Mixed-Integer Nonlinear Programming Models for Optimal Design of Multi-product Batch Plant, Fengqi You and Ignacio. E. Grossmann

Nonconvex MINLP Model

1. Nomenclature:

Sets/Indices

I	Set of products indexed by i
J	Set of processing stages site indexed by j
K	Set of digits of the binary representation indexed by k

Parameters

Q_i	Fixed production target of product i (kg)
S_{ij}	Size factor for product i in processing stage j (L/kg)
t_{ij}	Processing time for product i in processing stage j (hrs)
H	Fixed production horizon time (hrs)
α_j	Cost coefficient for processing stage j ($)
β_j	Cost exponent coefficient for processing stage j

Decision Variables (0 to $+\infty$)

V_j	Size of processing units required for stage j
N_j	Number of processing units required for stage j
B_i	Batch size of product i
TL_i	Cycle time of product i
z	Total investment cost ($)

Decision Variables (0-1)

Y_{kj}	Binary variable that determines the value of the *k*th digit of the binary representation of N_j.

2. Model Formulation:

Let V_j be the variable that represents the required volume of a unit in stage *j*, and B_i the variable that represents the size of the batch of product *i* at the end of all the stages. Since, the volume V_j has to be able to process all the products *i*, we have the following constraint for the volume of processing stage *j*:

$$V_j \geq S_{ij} B_i \,, \quad \forall i \in I \,, \quad j \in J \tag{1}$$

where the right hand side represents the actual volume needed by each product. The number of processing units N_j of stage *j* times the production cycle time TL_i of product *i* should be greater than the processing time, t_{ij}.

$$N_j TL_i \geq t_{ij} \,, \quad \forall i \in I \,, \quad j \in J \tag{2}$$

Since for product *i*, the required production amount is Q_i and the batch size is B_i, the number of batches is given by Q_i / B_i. Since the cycle time of each product is TL_i, the required processing time for product *i* is $Q_i \cdot TL_i / B_i$. Given the horizon time is H, the summation of the processing times of all the products should not exceed the horizon.

$$\sum_{i \in I} \frac{Q_i TL_i}{B_i} \leq H \tag{3}$$

Since the number of processing units processing units N_j should be an integer, we use the binary representation and a set of binary variables Y_{kj}, to model this variable.

$$N_j = 1 + \sum_{k \in K} 2^{k-1} Y_{kj} \,, \quad \forall j \in J \tag{4}$$

where $k \in K$ is the *k*th digit of a binary number and Y_{kj} is a binary variable that determines the value of the *k*th digit of the binary representation of N_j. The variable bounds are given as follows:

$$Y_{kj} \in \{0,1\} \,, \quad \forall j \in J \,, \quad k \in K$$

$$1 \leq N_j \leq N_j^U \,, \quad \forall j \in J$$

$$V_j^L \le V_j \le V_j^U \,,\ \ \forall j \in J$$

$$\max_j \left\{ \frac{t_{ij}}{N_j^U} \right\} \le TL_i \le \max_j \left\{ t_{ij} \right\},\ \ \forall i \in I$$

$$\frac{Q_i}{H} \cdot \max_j \left\{ \frac{t_{ij}}{N_j^U} \right\} \le B_i \le \min \left\{ Q_i, \min_j \frac{V_j^U}{S_{ij}} \right\},\ \ \forall i \in I$$

The objective function is to minimize the total investment cost, which is given by

$$z = \min \sum_{j \in J} \alpha_j N_j V_j^{\beta_j} \tag{5}$$

Therefore, we have the non-convex model formulation as follows:

$$\min\ z = \sum_{j \in J} \alpha_j N_j V_j^{\beta_j} \tag{5}$$

s.t.

$$V_j \ge S_{ij} B_i \,,\ \ \forall i \in I \,,\ \ j \in J \tag{1}$$

$$N_j TL_i \ge t_{ij} \,,\ \ \forall i \in I \,,\ \ j \in J \tag{2}$$

$$\sum_{i \in I} \frac{Q_i TL_i}{B_i} \le H \tag{3}$$

$$N_j = 1 + \sum_{k \in K} 2^{k-1} Y_{kj} \,,\ \ \forall j \in J \tag{4}$$

$$Y_{kj} \in \{0,1\} \,,\ \ \forall j \in J \,,\ \ k \in K$$

$$1 \le N_j \le N_j^U \,,\ \ \forall j \in J$$

$$V_j^L \le V_j \le V_j^U \,,\ \ \forall j \in J$$

$$\max_j \left\{ \frac{t_{ij}}{N_j^U} \right\} \le TL_i \le \max_j \left\{ t_{ij} \right\},\ \ \forall i \in I$$

$$\frac{Q_i}{H} \cdot \max_j \left\{ \frac{t_{ij}}{N_j^U} \right\} \le B_i \le \min \left\{ Q_i, \min_j \frac{V_j^U}{S_{ij}} \right\},\ \ \forall i \in I$$

This model is a non-convex mixed-integer nonlinear program (MINLP), and the non-convex terms are in the objective function and constraints (2) and (3). Due to the non-convexities, this model requires a global optimization solver to guarantee the global optimality of the solution.

Reference

1. Ignacio E. Grossmann, Roger W. H. Sargent, “Optimum Design of Multipurpose Chemical Plants,” Ind. Eng. Chem. Process Des. Dev., 2979, 18(2), 343-348
2. Gary R. Kocis, Ignacio E. Grossmann, “Global Optimization of Nonconvex Mixed-Integer Nonlinear Programming (MINLP) Problems in Process Synthesis”, Ind. Eng. Chem. Res., 1988, 27, 1407-1421