Solving Mixed-Integer Linear Fractional Programming Problems with Dinkelbach's Algorithm and MINLP Methods

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Problem Statement

A mixed-integer linear fractional program (MILFP) includes both continuous and discrete variables, and has linear constraints and an objective function given by the ratio of two linear functions. Mathematically, an MILFP can be formulated as the following problem (P):

$$\text{(P)} \quad \max \ \frac{a_0 + \sum_{i \in I} a_i x_i}{b_0 + \sum_{i \in I} b_i x_i}$$

$$\text{s.t.} \quad c_{0j} + \sum_{i \in I} c_{ij} x_i = 0, \quad \forall j$$

$$x_i \geq 0, \ \forall i \in I_1 \ \text{ and } \ x_i \in \{0,1\}, \ \forall i \in I_2 \ \text{ and } \ I_1 \cup I_2 = I$$

where it is assumed that $b_0 + \sum_{i \in I} b_i x_i > 0$ for all feasible solutions and where all inequalities are converted into equalities through the use of slack variables. Due to the nonlinearities in the objective function and the combinatorial nature because of the presence of 0-1 variables, the MILFP problems is a special type of mixed-integer nonlinear programming (MINLP) problems, which can become computationally intractable, especially for large scale instances with more than hundreds of binary variables.

Since (P) is also an MINLP problem, general optimization methods that can handle MINLP problems can be utilized for solving MILFP problems. Due to the non-convexity in the objective function, global optimal solutions of MILFP problems can be obtained by using global optimization methods. In addition, MINLP methods that can handle pseudoconvex/pseudoconcave objective function (eg. Branch & bound, α-ECP) are also able to globally optimize MILFP problems.[1] Other MINLP methods relying on convexity assumptions can be used to obtain optimal or near optimal solution quickly, although global optimality cannot be guaranteed.

Besides MINLP methods, Dinkelbach's algorithm can be also utilized to solve MILFP problems. Due to its optimality and convergence properties (such as super-linear convergence rate), we can use Dinkelbach's algorithm to globally optimize MILFP problems by iteratively solving a sequence of mixed-integer linear programming (MILP) problems.

In particular, we are interested in solving the following MILFP problems (CE) with the aforementioned two major approaches.

$$\text{(CE)} \quad \max \quad \frac{a_0 + \sum_{i \in I} a1_i x_i + \sum_{j \in J} a2_j y_j}{b_0 + \sum_{i \in I} b1_i x_i + \sum_{j \in J} b2_j y_j}$$

$$\text{s.t.} \quad c_{0k} + \sum_{i \in I} c1_{ik} x_i + \sum_{j \in J} c2_{jk} y_j = 0, \qquad \forall k \in K$$

$$b_0 + \sum_{i \in I} b1_i x_i + \sum_{j \in J} b2_j y_j \geq 0.001$$

$$0.1 \leq x_i \leq 1, \ \forall i \in I \ \text{ and } \ y_j \in \{0,1\}, \ \forall j \in J$$

Problem (CE) includes $|I|$ continuous variables, $|J|$ binary variables, and $|K|+1$ constraints/equations. The values of $|I|$, $|J|$ and $|K|$ range from 100 to 2,000. Given the large size of problems, the input data are generated randomly. a_0 is generated uniformly on U[-10, 10], b_0 is generated uniformly on U[-10, 10], $a1_i$ is generated uniformly on U[0, 10], $a2_j$ is generated uniformly on U[0, 10], $b1_i$ is generated uniformly on U[0, 10], $b2_j$ is generated uniformly on U[0, 10], c_{0k} is generated uniformly on U[-15, 20], $c1_{ik}$ is generated uniformly on U[-10, 10], $c2_{jk}$ is generated uniformly on U[-20, 15].

Reference

1. Fengqi You, Pedro M. Castro, Ignacio E. Grossmann, "Dinkelbach's algorithm as an efficient method to solve a class of MINLP models for large-scale cyclic scheduling problems," Computers & Chemical Engineering, In press (2009).