Shared Protection Network Design:
MILP and MINLP models

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Introduction. A well known problem in Telecommunication is to dimension the capacity on the arcs of a network so as to satisfy a given set of point-to-point traffic demands by providing one path from source to destination [2, 3]. An important subclass of problems is that of designing a network that is resilient to *single failures*.

One of the techniques that allow network resilience to single failures is *path protection*: two nodes s and t communicate through a single *working path* p from s to t, while a *backup path* p', *link disjoint* from p, i.e., that has no edge in common with p, is is used upon failure of any of the arcs of p. Traffic is routed on p' when an edge of p goes out of order. This ensures network redundance to only one failed edge at a time; however, the possibility of re-establishing a link soon after failure makes this *single link failure* hypothesis acceptable in most cases.

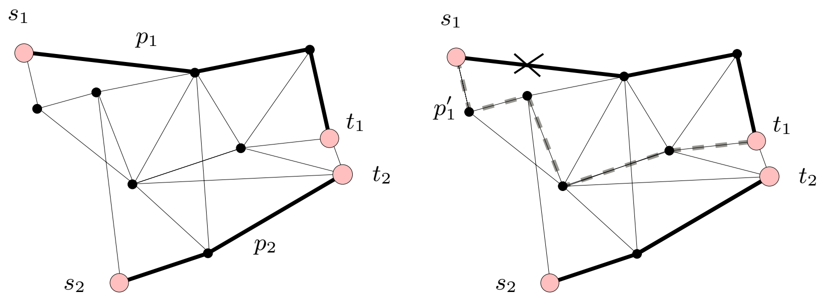
Although backup paths are only used upon link failure, they are allocated part of the capacity and hence take part in the network cost. Within a standard path protection scheme, each backup path is exclusively associated part of the edge capacity. However, in the single failure hypothesis there are pairs of demands whose backup paths are not used simultaneously, in any failure scenario. As described below, in *Shared Protection* (SP) part of the installed capacity can be shared among backup paths if they are not used simultaneously in the event of a failure [1].

Setting. Consider a network topology defined as a digraph (V, A) and a set Q of point-to-point traffic demands (also termed *commodities*) described by triples (s_q, t_q, d_q), $q \in Q$, standing for the origin, destination, and traffic volume requested for a commodity q. Consider two demands between node pairs (s_1, t_1) and (s_2, t_2), as in Fig. 1a. In nominal conditions, they are routed on paths p_1 and p_2, which share no edge. If a failure occurs on an edge in p_1, network management routines keep the connection active through a backup path p'_1, as highlighted in Fig. 1b. Similarly, if a failure occurs on p_2 a backup path p'_2 replaces the nominal one (see Fig. 1c).

Backup paths are necessary to network survivability, yet increase remarkably the network cost: each traffic demand contributes twice to the total capacity installed, even though the backup resources are only used upon failure. If the two nominal paths p_1 and p_2 have no edge in common (as in Fig. 1) and under the single failure hypothesis, utilization of the two backup paths p'_1 and p'_2 is mutually exclusive. Hence, these paths can be assigned the same resources on all the edges they share, thus reducing the capacity installed to these two node pairs on $p'_1 \cap p'_2$, which is $\max(d_{q_1}, d_{q_2})$ rather than $d_{q_1} + d_{q_2}$ (see Fig. 1d).

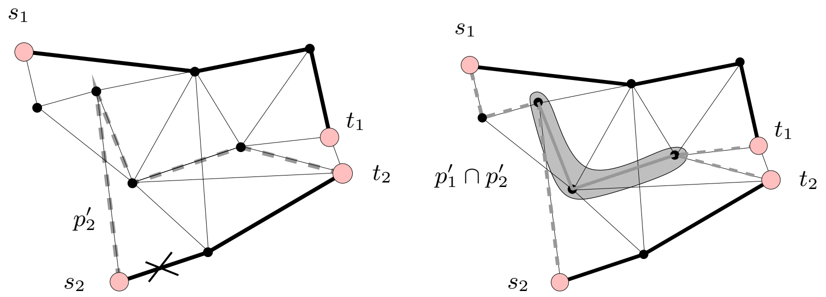
In other words, SP allows to share backup resources and therefore leads to a cheaper network when compared to *Dedicated Protection*, which reserves capacity for every backup path.

Definition of the problem. Given a unitary capacity cost $c : A \to \mathbb{R}$ for each arc and a set Q of triplets (s_q, t_q, d_q) denoting source, destination, and volume of traffic of a point-to-point demand, install capacity on the arcs of (V, A) so that all demands are satisfied even in the case of a single failing link. Only two paths between source and



(a) Nominal condition

(b) Failure on path p_1



(c) Failure on path p_2

(d) Shareable resources

Fig. 1. Shared backup resources in a network with path protection.

destination are allowed: one (the *working path*) when no failure occurs and one when one (the *backup path*) when one of the arcs of the working path is broken. Minimize the total network cost, defined by the sum, over all arcs, of the unitary capacity cost times the amount of capacity to be installed on that arc.

Model. Given a network topology (V, A) and traffic demands (s_q, t_q, d_q), $q \in Q$, the solution is a subset of A with associated capacity such that a routing of all demands is performed and the cost of the topology is minimum. The set of arcs A induces a set E of ordered node pairs: $E = \{(i,j) \in V^2 : i < j \land ((i,j) \in A \lor (i,j) \in A)\}$. Each arc (i,j) is associated a cost c_{ij}.

Integer variables Y_{ij} define the capacity to be installed (i.e., the total traffic data) on arc (i,j). Routing paths are defined by binary variables F_{ij}^q, which describe the working flow on each arc (i,j) for every demand q. The binary variable G_{ij}^q identifies instead the backup data flow on arc (i,j) for demand q. These classes of variables are subject to flow conservation constraints, imposing a unitary unsplit flow to follow a path from s_q to t_q:

$$\begin{array}{ll} \sum_{j \in V:(i,j)\in A} F_{ij}^q - \sum_{j \in V:(j,i)\in A} F_{ij}^q = b_{iq} & \forall i \in V, q \in Q \\ \sum_{j \in V:(i,j)\in A} G_{ij}^q - \sum_{j \in V:(j,i)\in A} G_{ij}^q = b_{iq} & \forall i \in V, q \in Q \end{array}$$

with $b_{iq} = \begin{cases} \;\;1 \text{ if } i = s_q \\ -1 \text{ if } i = t_q \\ \;\;0 \text{ otherwise.} \end{cases}$

Moreover, link disjointness must hold between working and backup flows:

$$F_{ij}^q + F_{ji}^q + G_{ij}^q + G_{ji}^q \leq 1 \qquad \forall (i,j) \in E.$$

In dedicated (non-shared) protection, the capacity to be installed on arc (i,j) is equal to the overall data flow, i.e. $Y_{ij} = \sum_{q\in Q} d_q(F^q_{ij} + G^q_{ij})$, and can be discerned as working and backup capacity. In Shared Protection, the total working capacity on arc (i,j) is still $\sum_{q\in Q} d_q F^q_{ij}$, but the backup capacity, denoted as ξ_{ij}, may decrease as pointed out above. Let us consider a failure on an edge[1] (h,k) different from arc (i,j). The capacity required by commodity q on (i,j) due to this failure is d_q if the working path of q contains arc (h,k) or arc (k,h) and its backup path contains (i,j), or

$$G^q_{ij} = 1 \ \wedge \ (F^q_{hk} = 1 \vee F^q_{kh} = 1).$$

Notice that the two terms in parentheses are mutually exclusive: it is easily shown that a routing path does not cross an edge in both directions. We introduce variables $Z^q_{ij,hk}$ for every triple $q \in Q$, $(h,k) \in E$, and $(i,j) \in A$ such that $(i,j) \neq (h,k)$ and $(j,i) \neq (h,k)$. Variable $Z^q_{ij,hk}$ is one if commodity q needs to route a backup flow on arc (i,j) in case of failure on edge (h,k), zero otherwise:

$$Z^q_{ij,hk} \ = \ G^q_{ij}(F^q_{hk} + F^q_{kh}),$$

but does not need to be binary because F and G are binary and because of the above constraint. The value of ξ_{ij} can be found by considering every condition of failure: the backup flow due to failure of edge (h,k) is $\sum_{q\in Q} d_q Z^q_{ij,hk}$, and the capacity needed to cope with all possible failure situations is the maximum over all edges (i.e., failures), $\xi_{ij} = \max_{(h,k)\neq(i,j)} \sum_{q\in Q} d_q Z^q_{ij,hk}$. Hence, Y_{ij} is the maximum total traffic under all failure situations:

$$\forall (i,j) \in A \qquad Y_{ij} = \max_{(h,k)\neq(i,j)} \sum_{q\in Q} d_q \left(f^q_{ij} + Z^q_{ij,hk} \right).$$

Three models for SPND are in the remainder of this submission. The sum of deployment and capacity costs

$$\sum_{(i,j)\in A} c_{ij} Y_{ij}$$

is minimized subject to flow conservation constraints for F and G, to the definition of capacity Y, to link disjointness between F and G, and finally to the definition of Z as a function of F and G.

References

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2. A.A. Assad, Multicommodity Network flow – a survey, Networks 8 (1978), pp. 37-91.
3. D. Yuan. (2001). “An annotated bibliography in communication network design and routing”, in *Optimization models and methods for communication network design and routing*, PhD dissertation no. 682, Institute of Technology, Linköpings Universitet (Sweden).

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[1] Failures in Telecommunication networks usually disrupt communication in both directions of a link, and are usually identified by edges rather than arcs.