Stochastic Portfolio Optimization with Round Lot Trading Constraints

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1 Problem Description

The mean-variance approach proposed by Markowitz studies how investors can construct optimal portfolios taking into consideration the trade-off between market volatility and expected returns. Out of a universe of r risky assets and one non-risky asset characterized by a known return μ_0 that usually reflects the interest rate on the money market, an efficient frontier of optimal portfolios can be constructed. Portfolios on the efficient frontier offer the maximum possible expected return for a given level of risk. The original Markowitz model assumes that the expected returns $\bar{\mu} \in \mathcal{R}^r$ of the risky assets and the variance-covariance matrix $\Sigma \in \mathcal{R}^{r \times r}$ of the returns are known. One of the several formulations of the mean-variance portfolio selection problem involves the construction of a portfolio with minimal risk provided that a prescribed return level R is attained (2), and is formulated by the mathematical program:

$$\min \quad w^T \Sigma w \tag{1}$$

$$\text{subject to} \quad \mu_0 w_0 + \bar{\mu}^T w \geq R \tag{2}$$

$$\sum_{j=0}^{r} w_j = 1 \tag{3}$$

$$w \in \mathcal{R}^{r+1} \tag{4}$$

The decision variables w_j, $j = 1, \ldots, r$ represent the proportion of capital invested in the risky asset j, while w_0 is the fraction of capital invested in the money market. The objective function (1) minimizes the variance of the portfolio $w^T \Sigma w$, and the constraint (3) enforces that the sum of the investments is equal to 1. The investor can allocate part (w_0) of the available capital K to the money market. Short-selling is here not allowed (4).

Much effort has been devoted to extending Markowitz work and making the modern portfolio theory more practical. Two of the major limitations of the mean-variance approach are that it does not account for *(i)* trading restrictions such as the need to buy stocks in large lots or purchases [1], the presence of transaction costs, etc., and for *(ii)* the randomness in the parameters describing the model [1, 2].

References

[1] Bonami P., M.A. Lejeune. 2009. An Exact Solution Approach for Integer Constrained Portfolio Optimization Problems under Stochastic Constraints. *Operations Research* 57 (3), 650-670.

[2] M.A. Lejeune. A VaR Black-Litterman Model for the Construction of Absolute Return Fund-of-Funds. Forthcoming in *Quantitative Finance*.