Simultaneous Cyclic Scheduling and Control of a Multiproduct CSTR

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Results and Discussion

The application of the optimization formulation will be illustrated using a CSTR where the following simple irreversible reaction

$$3R \xrightarrow{k} P, \; -\mathcal{R}_R = kC_R^3$$

takes place for manufacturing 5 products A, B, C, D, E. The dynamic composition model is given by,

$$\frac{dC_R}{dt} = \frac{Q}{V}(C_o - C_R) + \mathcal{R}_R \tag{1}$$

where C_o stands for feed stream composition and Q is the control variable for the dynamic transition in the production of one product to another. Using the following values of the design and kinetic parameters: $C_o = 1$ mol/lt, $V = 5000$ lt, $k = 2$ lt^2/(mol^2-h), and the five values of the volumetric flowrate Q shown in Table 1, the concentration five steady-states C_R, shown in the same table, are obtained. All the examples featured in this work require the steady-states values of the manipulated (u) and controlled variables (x) for manufacturing each one of the products. In addition, Table 1 also features values of the demand rate (D_i), product cost (C_i^p) and inventory cost (C_i^s).

Solving the MIDO scheduling and control problem using GAMS/DICOPT, the

Product	Q [lt/hr]	C_R [mol/lt]	Demand rate [Kg/h]	Product cost [$/kg]	Inventory cost [$]
A	10	0.0967	3	200	1
B	100	0.2	8	150	1.5
C	400	0.3032	10	130	1.8
D	1000	0.393	10	125	2
E	2500	0.5	10	120	1.7

Table 1: Process data for the first case study. A, B, C, D and E stand for the five products to be manufactured. The cost of the raw material (C^r) is $10.

Slot	Product	Process time [h]	Production rate [Kg/h]	w [Kg]	Transition Time [h]	T start [h]	T end [h]
1	A	41.5	9.033	374.31	5	0	46.4
2	E	23.3	1250	29162.3	5	46.4	74.7
3	D	2.06	607	1247.7	5	74.7	81.8
4	C	4.48	278.72	1247.7	5	81.8	91.2
5	B	12.48	80	998.2	21	91.2	124.7

Table 2: Simultaneous scheduling and control results for the first case study. The objective function value is $7889 and 124.8 h of total cycle time.

optimizer selects the cyclic $A \to E \to D \to C \to B$ production wheel as the one which maximizes the profit. The objective function value turned out to be $7889, while the total cycle time was 124.8 h. Additional information concerning processing times at each slot, production rates, total amounts of each product, transition times and initial and ending times at each slot are shown in Table 2. Regarding the dynamic behavior of the reactor during product transitions, Figure 1 displays the dynamic profiles of both the manipulated variable (Q) and the controlled variable (C_R).

It is interesting to compare the optimal MIDO solution against the second and third best cyclic solutions. Moreover, in order to compare the performance of DICOPT++ when solving MIDO problems, the second and third best optimal solutions of all the examples were always computed using SBB (other MINLP solver available in GAMS). In the present example, the second best solution, which was obtained by adding an integer cut, is in fact a slight variation of the previous one. In this case the optimizer selects the cyclic $A \to D \to E \to C \to B$ processing sequence with profit \$7791 and a cycle time of 125 h. To learn the reasons why the first production sequence turns out to be better than the second one, we have partitioned the objective function in three terms: ϕ_1 dealing with the product profits, ϕ_2 which deals with the inventory costs and ϕ_3 dealing with the transition costs and defined as follows,

$$\phi_1 = \sum_{i=1}^{N_p} \frac{C_i^p W_i}{T_c} \tag{2}$$

$$\phi_2 = \sum_{i=1}^{N_p} \frac{C_i^s (G_i - W_i)}{2\Theta_i T_c} \tag{3}$$

$$\begin{aligned} \phi_3 = & \sum_{k=1}^{N_s} \sum_{f=1}^{N_{fe}} h_{fk} \sum_{c=1}^{N_{cp}} \frac{C^r t_{fck} \Omega_{c,N_{cp}}}{T_c} ((x_{fck}^1 - \bar{x}_k^1)^2 + \ldots + \\ & (x_{fck}^n - \bar{x}_k^n)^2 + (u_{fck}^1 - \bar{u}_k^1)^2 + \ldots + (u_{fck}^m - \bar{u}_k^m)^2) \end{aligned} \tag{4}$$

hence, the $[\phi_1, \phi_s, \phi_3]$ values are [32397, 23262, 1247] and [32463, 23330, 1234] for the first and second solutions, respectively (see Table 3 for information regarding optimal values of the additional decision variables). From this information, we see that both solutions have similar ϕ_1 and ϕ_2 values. However, the difference between those solutions is the transition cost: the second solution features a larger transition costs and this makes it suboptimal compared to the first one. Dynamic product transitions for this production sequence are depicted in Figure 2. As can be seen, the dynamic product transitions feature a shape that resembles the results of the best MIDO solution.

Regarding the third best MIDO optimal solution, the optimizer selected the cyclic $B \to A \to E \to C \to D$ production sequence with profit \$6821.6 and cycle time of 127 h. Information about the decision variables of this solution can be found in Table 4. As we can see, the third optimal solution has a larger objective function value decrease compared to the second one. Analyzing the $[\phi_1, \phi_2, \phi_3] = [31967, 23352, 1794]$ values we notice that the third solution features a decrease in ϕ_1 (the profit associated to product manufacture is smaller) and in increase in ϕ_3 (larger transition cost). This

Slot	Product	Process time [h]	Production rate [Kg/h]	w [Kg]	Transition Time [h]	T start [h]	T end [h]
1	A	41.5	9.033	374.31	5	0	46.4
2	D	2.06	607	1249.4	5	46.4	53.6
3	E	23.4	1250	29270.4	5	53.6	82
4	C	4.48	278.72	1249.4	5	82	91.5
5	B	12.48	80	999.5	21	91.5	125

Table 3: Simultaneous scheduling and control results for the first case study, second best solution. The objective function value is \$7791 and 125 h of total cycle time.

Slot	Product	Process time [h]	Production rate [Kg/h]	w [Kg]	Transition Time [h]	T start [h]	T end [h]
1	B	12.7	80	1012.5	21	0	33.7
2	A	42.04	9.033	379.7	5	33.7	80.7
3	E	23.3	1250	29125.4	5	80.7	109
4	C	4.6	278.72	1265.6	5	109	118.6
5	D	2.09	607	1265.6	6	118.6	127

Table 4: Simultaneous scheduling and control results for the first case study, third best solution. The objective function value is \$6821.6 and 127 h of total cycle time.

cost combination makes this production sequence worse than the first and second ones, even though the shape of the dynamic transitions looks similar to the first and second cases as depicted in Figure 3. The CPU times (IBM Laptop, 1.6 Ghz, Windows XP) needed for MIDO problem solution were 13.8, 67 and 27.8 secs for the best, second and third solutions, respectively.

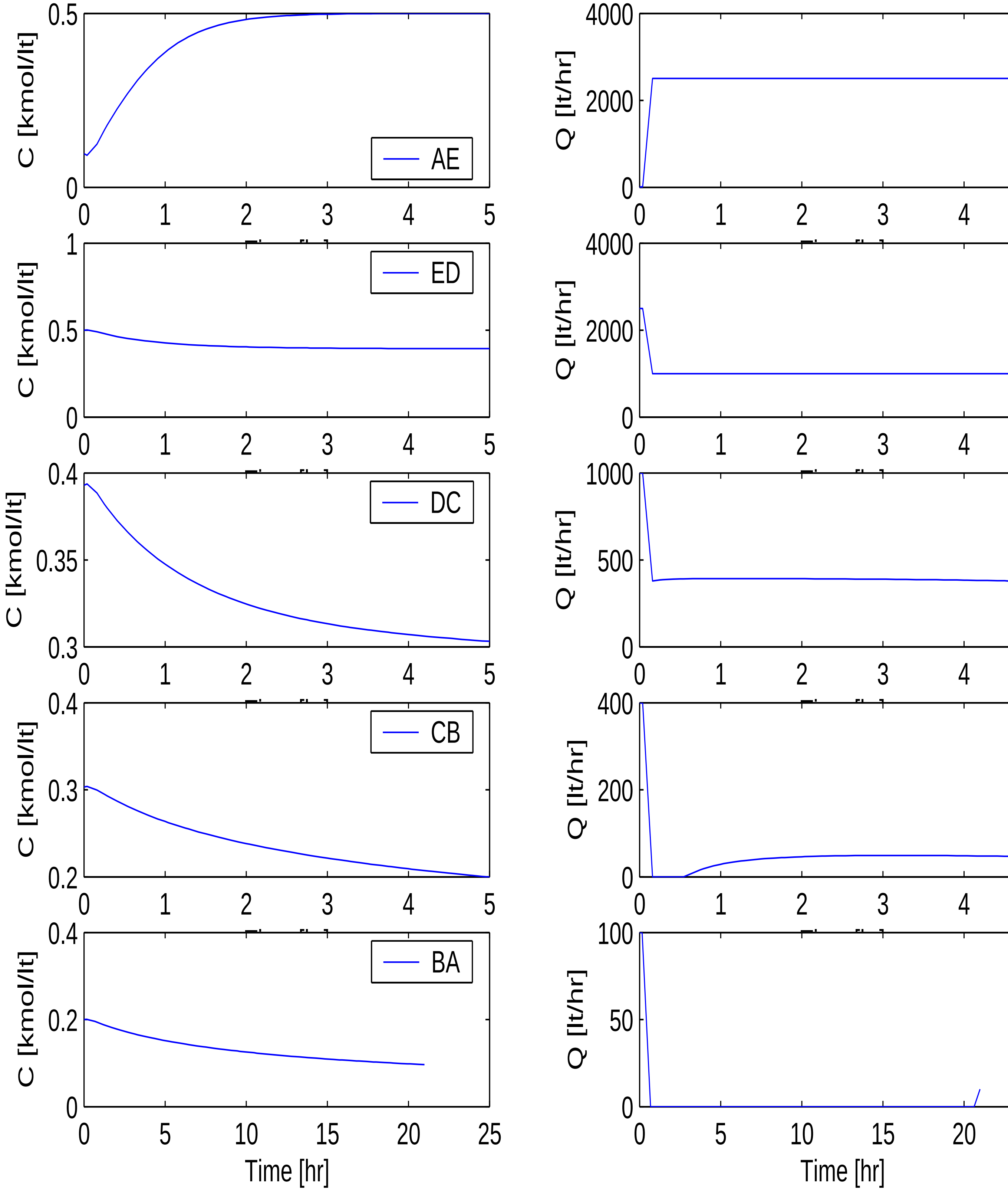

Figure 1: Optimal dynamic profiles for the volumetric flow rate and reactor concentration during product transition for the first case study.

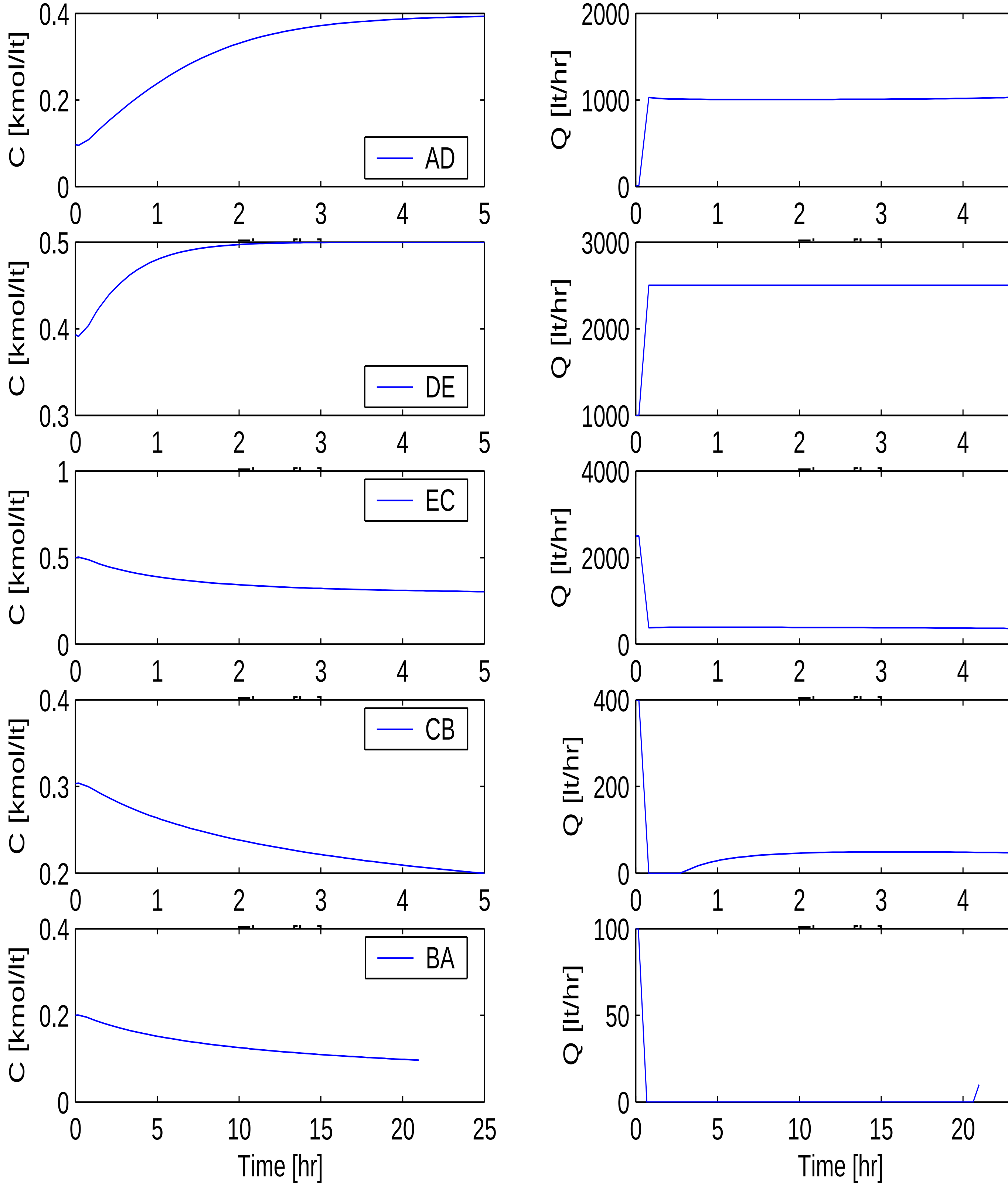

Figure 2: Optimal dynamic profiles for the volumetric flow rate and reactor concentration during product transition for the first case study, second best solution.

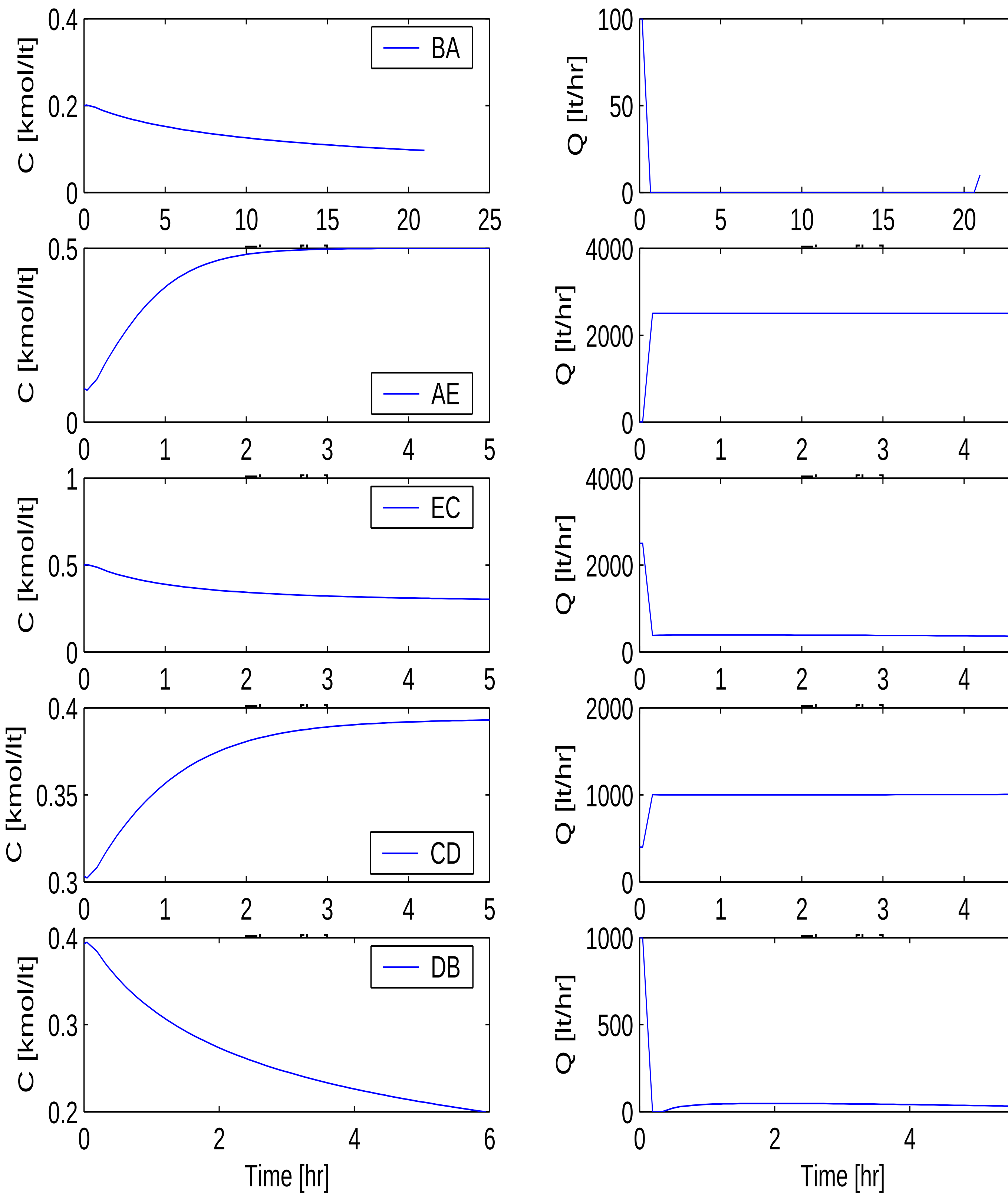

Figure 3: Optimal dynamic profiles for the volumetric flow rate and reactor concentration during product transition for the first case study, third best solution.