MPEC strategies for optimization of pipeline operations

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Results

A large network with 3 sources, 15 demands, 29 arcs, and 30 nodes was defined as shown in Figure 1. This fictitious network is based loosely on the actual pipeline network described in [9]. All arcs were assumed to have a diameter of 10 cm, and roughness of $\epsilon_i = 50$ microns. Physical properties of oxygen were used. The sources used an ambient temperature of 298.15K and a compressor efficiency of 85%.

Energy minimization

The objective function for the energy minimization problem is as described above:

$$\begin{aligned} Min \quad & \sum_{s \in S} \int_{t=0}^{t=t_{end}} Power_{s,t} dt + 10^{-6} \sum_{s \in S} \sum_{t \in T \setminus \{0\}} (Power_{s,t} - Power_{s,t-1})^2 \\ s.t. \quad & \text{Equations (1)-(21)} \end{aligned}$$

The regularization term can be seen as a tuning factor to address various operational concerns. A small regularization will only slightly smooth the compressor profiles and may lead to excessive wear on the process equipment or other operational problems not easily included in a mathematical programing context. In contrast a larger regularization will significantly smooth the compressor profiles, but may lead to excessively conservative and suboptimal operation.

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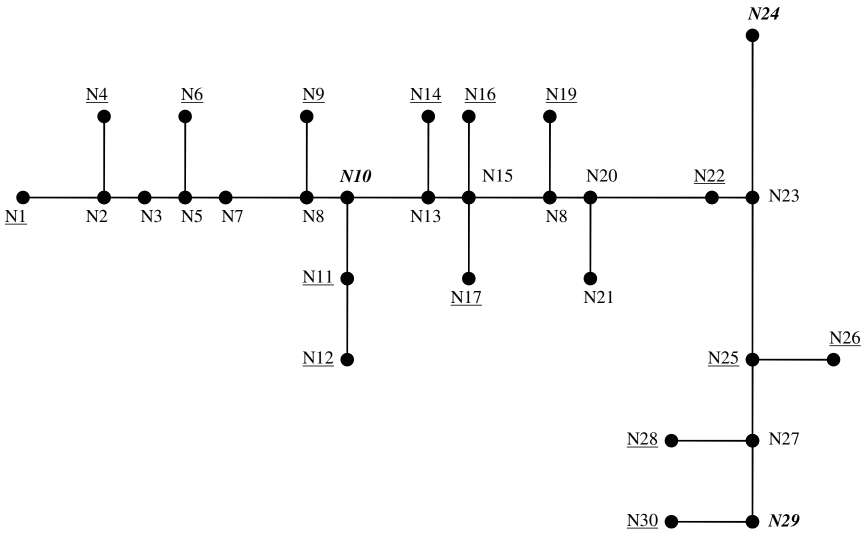

Figure 1: Large network used in the case study. Pipe segment lengths are shown in km, nodes are labeled N#, with source nodes (suppliers) labeled in bold italics, and demand nodes (customers) underlined.

The energy minimization problem is formulated as an NLP by approximating the time integral and differential equations with the trapezoidal rule. The problem was made arbitrarily large by increasing the number of 1 hour time periods considered. The problems were initialized at steady state solutions for average customer demand. However, customer demand was assumed to be sinusoidal with a period of 2π hours and an amplitude of 5% of the nominal value. Furthermore, a final time constraint was added to the problem. In particular, the inventory in the network at final time must be at least as large as the inventory in the network at initial time.

$$mass_{i,t} = \frac{\bar{P}_{i,t} MWA_i L_i}{RT_{ref}} \qquad \forall i \in I, t \in T \tag{1}$$

$$\sum_i mass_{i,0} \leq \sum_i mass_{i,t_f} \tag{2}$$

Results are presented in Table 1 for different problem sizes considered. The number of iterations required and computational time increase with the problem size and more details regarding the computational scaling of the problem can be found in [3].

Time Periods	Iterations	Solution Times [s]	Variables	Equations	Complementarity Constraints
10	517	38.403	14858	14621	2552
50	2171	932.089	68778	67781	11832
100	4477	3893.151	136178	134231	23432

Table 1: Large case study solution results. Solution times, variable, equation, and complementarity constraint counts are presented.

The above problem can also be modified to include economics objectives for various energy pricing schemes. More details can be found in [3].

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