

RESULTS AND DISCUSSION

A real-world case study involving the transportation of six different oil derivatives (P1-P6) by pipeline from a unique oil refinery to a single distribution center has been tackled. This unidirectional multiproduct pipeline has a length of 147 km and a capacity of 18,000 v.u. (volumetric units), and it carries four liquid products and two liquefied gases. The oil refinery and the pipeline are both owned and operated by the same company. The tank farm at the distribution center comprises dedicated storage tanks for liquid products and spheres for gases. In order to fill up exactly either one or multiple tanks, the batch size for each product can be selected from a limited number of size options, i.e. at most three per product. The maximum storage capacity is also given.

The pumping rate of any product is assumed to be fixed at 519.4 v.u./h (volumetric units per hour). Moreover, maintenance shutdown periods are not expected over the planning horizon. Client demands are known by the refiner two weeks before the beginning of each month. Enough product inventories at refinery tanks are assumed to be available to fulfill the pipeline pumping run schedule. Therefore, no control of product inventory levels in refinery tanks will be needed. For simplicity, interface volumes are neglected. Nonetheless, some product sequences are forbidden. For instance, P5 is the only product that can be injected immediately before P6 while P1 can merely be preceded by either P2, P3 or P4. A minimum settling time of 24 h for product certification has been adopted. Daily product demands are to be satisfied and, therefore, removed from the available inventories at the start of every day. The monthly product demand is supposed to be uniformly distributed along the month with some few exceptions. In fact, there is no demand for P3 and P5 at some particular day of every week.

The pipeline is initially full of product P1. The input data for this case study is given in Tables 1, 2 and 3. For all the examples, the objective function attempts to minimize the overall backorder cost due to tardily fulfilling daily market demands.

Table 1. Maximum storage capacities and alternative lot sizes for each product

Product (p)	$ID_{\max,p}$ (v.u.)	Alternative lot sizes (in v.u.)		
		Option 1	Option 2	Option 3
P1	81,500	21,800	18,000	17,300
P2	32,000	16,000	8,000	--
P3	24,000	16,000	8,000	--
P4	27,800	16,000	8,000	3,800
P5	10,320	3,440	1,720	860
P6	13,120	6,560	4,920	8,200

Table 2. Allowed preceding products for each oil derivative

	Allowed preceding products					
	P1	P2	P3	P4	P5	P6
P1		✓	✓	✓		
P2	✓					
P3	✓			✓	✓	
P4	✓		✓		✓	
P5			✓	✓		✓
P6					✓	

Table 3. Monthly demand and initial inventory for each product (in v.u.)

Product	P1	P2	P3	P4	P5	P6
Monthly Demand (v.u.)	198,043	64,800	14,642	68,244	10,934	16,955
Initial Inventory (v.u.)	52,397	17,565	18,569	19,888	10,027	7,309

In real-life operations, the product sequence generally follows some particular batch cycle. Such a sequencing pattern often results from systematic practical approaches developed by schedulers to avoid undesired product contamination. However, the product sequence is a key scheduling decision to be established by solving the proposed formulation. In order to compare the performance of our pipeline scheduling approach, three different instances of the case study previously defined have been considered. They will be called Examples 1, 2 and 3. In Example 1, the sequence of products to be shipped through the pipeline is arbitrarily adopted by the scheduler (i.e. a pre-fixed product sequence)

before solving the problem formulation to just optimize the lot sizing process. Example 2 assumes that the pipeline scheduler has adopted an incomplete pre-defined product sequence with a limited number of open positions to be filled with pre-defined allowable products. In this case, the problem model is aimed at optimally assigning products to open positions and determining the size of every batch. In Example 3, the proposed formulation permits to establish both the complete sequence of products to be injected in the pipeline and the lot sizing.

For all the problem instances, we adopted as stopping criterion a maximum resource time of 20000 CPU seconds or a final solution within a maximum relative tolerance of 0,1 %. We solved the problem with the MILP models directly by using CPLEX 11.2.1 and GUROBI 1.0.4 solvers with GAMS 23.0.2 on an Intel Core i7 2.93 GHz machine with 4 processors and 8 GB RAM.

Example 1

In the first instance of the case study, the sequence of product lots to be pumped in the pipeline is adopted by the scheduler before solving the problem, i.e. the fixed sequence case. Model size, including the number of binary/continuous variables and constraints in the problem formulation, are shown in Table 4.

Table 4. Comparison of model sizes for Examples 1, 2 and 3

	Model Type		
	fixed sequence	mixed sequence	free sequence
Constraints	5307	5331	6526
Continuous variables	5210	5491	11172
Binary variables	2046	2063	2464

The optimal product sequence and volumes of batch injections (given in 10^3 v.u.) are presented in Table 5. For the sake of comparison, it is also given the input schedules obtained for Examples 2 and 3.

Table 5. Fixed, mixed and free product sequences found for Examples 1, 2 and 3

Example	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15	I16	I17	I18
1	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2
	21,8	16	21,8	8	0,86	4,92	0,86	8	21,8	8	21,8	8	0,86	4,92	0,86	8	21,8	16
2	P1	P2	P1	P4	P5	P6	P5	P3	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2
	21,8	16	21,8	16	1,72	4,92	0,86	16	21,8	16	21,8	8	1,72	4,92	0,86	8	21,8	16
3	P1	P2	P1	P4	P3	P4	P1	P2	P1	P4	P5	P6	P5	P6	P5	P4	P1	P2
	21,8	16	17,3	3,8	8	16	17,3	8	17,3	3,8	0,86	6,56	1,72	6,56	0,86	8	21,8	16

Example	I19	I20	I21	I22	I23	I24	I25	I26	I27	I28	I29	I30	I31	I32	I33	I34	I35
1	P1	P4	P5	P6	P5	P4	P1	P2	P1	P3	P5	P6	P5	P4	P1	P2	P1
	21,8	8	0,86	4,92	0,86	8	21,8	16	21,8	16	0,86	4,92	3,44	16	21,8	16	21,8
2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1
	21,8	8	1,72	4,92	0,86	16	21,8	16	21,8	3,8	0,86	6,56	1,72	3,8	21,8	8	21,8
3	P1	P4	P1	P2	P1	P4	P3	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2
	21,8	16	21,8	8	17,3	3,8	8	21,8	16	21,8	16	3,44	8,2	3,44	3,8	17,3	8

Table 6. Comparison of the model performance for Examples 1, 2 and 3

	MILP Solver					
	GUROBI 1.0.4			CPLEX 11.2.1		
	fixed sequence	mixed sequence	free sequence	fixed sequence	mixed sequence	free sequence *
Obj. Fun.	1541599	1526899	1478599	1541599	1526899	3232422
Pipe Usage (%)	97.6	98.2	100.0	97.6	98.2	70.4
CPU (s)	83.4	1511.2	20000	125.2	20000	20000
Iterations (10 ⁶)	1.0	17.9	55.4	3.5	231.7	55.9
Relative Gap (%)	0.0058	0.0084	0.0443	0.0058	0.1556	54.2775

* Non-optimal

Example 2

In the second instance of the case study, the assignment of product P3 or P4 to open positions on the base product sequence is left to the model. This is the so-called mixed product sequence case since part of the product sequence has been pre-fixed and the remaining one is selected by the model. Despite the

problem size for the mixed-sequence case slightly increases with regards to Example 1, the required CPU time and the number of nodes to be explored to ensure a relative gap lower than 10^{-4} show a significant increase with respect to Example 1. The optimal mixed-sequence is rather the same as the fixed case, except for an earlier injection of product P3, that is anticipated from the batch injection number 28 to 8. Both solvers obtain the same solution, but CPLEX is not able to guarantee optimality after 20000 seconds of CPU time. Compared with the fixed sequence case, the pipeline usage raises from 97.6 to 98.2 %.

Example 3

In the free-sequence case, the complete product sequence and the lot sizes are all optimized through the proposed MILP mathematical model. In this way, a non-cyclic schedule significantly different from the ones found in Examples 1-2 has been obtained. However, some product sub-sequences adopted in the previous examples still remain in the optimal solution like (P4-P1-P2-P1-P4) and (P1-P4-P5-P6-P5). The number of lots of P1 and P4, i.e. the products with the largest monthly load, increases from 10 to 11 and from 7 to 8, respectively. Moreover, there is an additional lot of product P2 and another of product P3. All at the expense of diminishing the batches of P5 and P6 by 3 and 1, respectively. The pipeline usage finally rises to 100 %.

The only solver that could find the optimal free-sequence solution is GUROBI. Nonetheless, after 20000 of CPU time, the relative gap could not be reduced below 0.04 %. Besides, after 55.4 million iterations, the initial lower bound given by the relaxed LP solution was not improved. GUROBI found an integer solution quite close to the relaxed one. On the other hand, CPLEX could not find a reasonable feasible solution after 20000 CPU seconds.