Integrated Process Water Networks Design Problem

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Results and Discussions

The application of this model is illustrated first with a small example problem for an integrated water network with 2 process units (PU), 2 treatment units (TU), 2 contaminants (A, B) and a single source of water (see superstucture given in Figure 2).

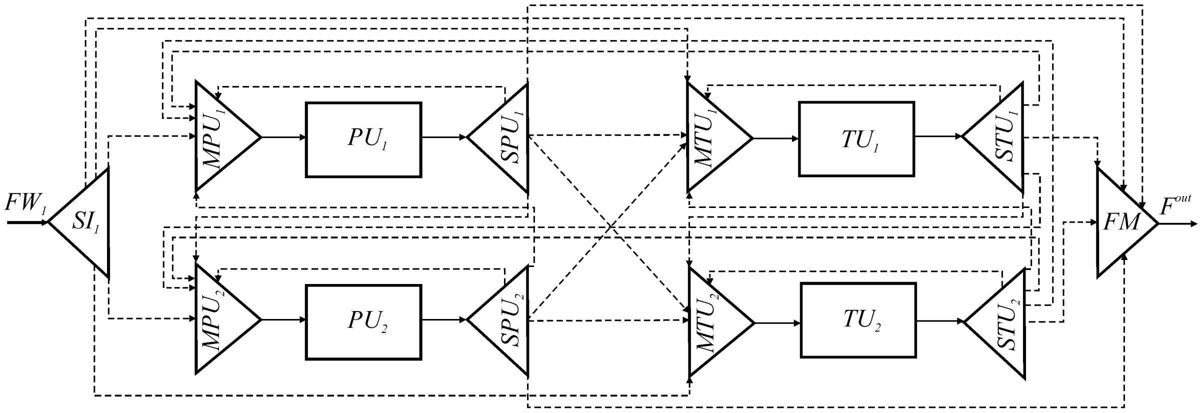

Figure 2. Water network superstucture for the example problem 1.

The input data of this example is given in Tables 1 and 2. Each treatment unit can remove only one contaminant. The environmental discharge limit for contaminant A and contaminant B is 10 ppm.

Table 1. Data of process units for the example problem 1.

Process unit	Flowrate (t/h)	Discharge load (kg/h)		Maximum inlet concentration (ppm)	
		A	B	A	B
PU_1	40	1	1.5	0	0
PU_2	50	1	1	50	50

The freshwater cost is assumed to be $1/ton, the annualized factor for investment is taken to be 0.1, and the total time for the network operation in a year is assumed to be 8000 h. The fixed cost pertaining to the pipes is assumed to be $6, the variable cost for each individual pipe $100, and operating cost coefficient for pumping water through pipes $0.006/ton. The tolerance selected for the optimization with BARON and LindoGlobal was zero.

Table 2. Data of treatment units the example problem 1.

Treatment unit	% removal of contaminant		IC (Investment cost coefficient)	OC (Operating cost coefficient)	α
	A	B			
TU_1	95	0	16800	1	0.7
TU_2	0	95	12600	0.0067	0.7

The non-convex MINLP formulation for this example contains 84 constraints, 20 binary variables, and 75 continuous variables. We solved this problem in GAMS with the non-convex MINLP model directly by using the BARON 8.1.5 and LindoGlobal 23.0.2 solver on a Toshiba Satellite Notebook with 4 GB RAM memory with Intel Core Duo 2 processor. The objective function was to minimize the total network cost. The optimal water network design for this problem is given in Figure 3, and the optimal objective function values and the computational times in Table 3.

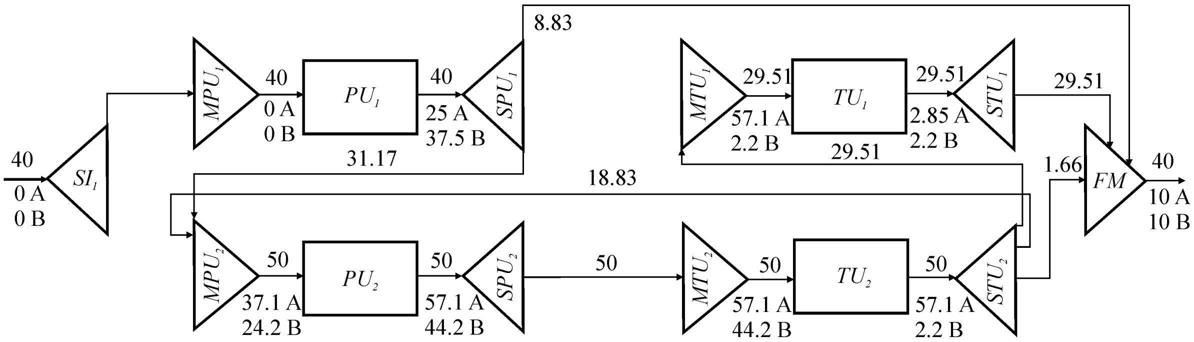

Figure 3. Optimal solution of water network design for the example problem 1.

In both cases, the optimal objective function value ($606760.55/year) as well as the network design (see Figure 3) is the same. However, LindoGlobal solved the non-convex MINLP model in less computational time compared to BARON (10.30 CPUs vs. 64.30 CPUs). It should be noted when solving this model with BARON the derivatives become unbounded if the flows take zero values in the cost terms of the objective function. Hence, we added a tolerance $\varepsilon = 0.001$ in the cost function ($\text{Cost} = a\cdot(\text{Flow}+\varepsilon)^{b}$) for the investment cost of the pipes in the network and for treatment units. While this introduces a small error (see Table 3) in the objective function, it leads to bounded gradients for zero flows, and the computational time for BARON in this case is less expensive (CPU is reduced from 64.30 CPUs to 3.4 CPU) compared to LindoGlobal (3.4 CPUs vs. 10.30 CPUs). This in an important issue for complex water netwoks problems as will be shown in the second example.

Table 3. Computational performance of the Non-convex MINLP model for the example problem 1.

Solver	without ε		with $\varepsilon = 0.001$		Error (%)
	Objective function	CPU (s)	Objective function	CPU (s)	
BARON	606760.55	64.30	606763.17	3.40	$4.31 \cdot 10^{-4}$
LindoGlobal	606760.55	10.30	606763.17	10.90	$4.31 \cdot 10^{-4}$

The second example problem is more complex than the first one. In this case, an integrated process water network consists of 5 process units (PU), 3 treatment units (TU), 3 contaminants (A, B, C) and a single source of water (see superstucture given in Figure 4).

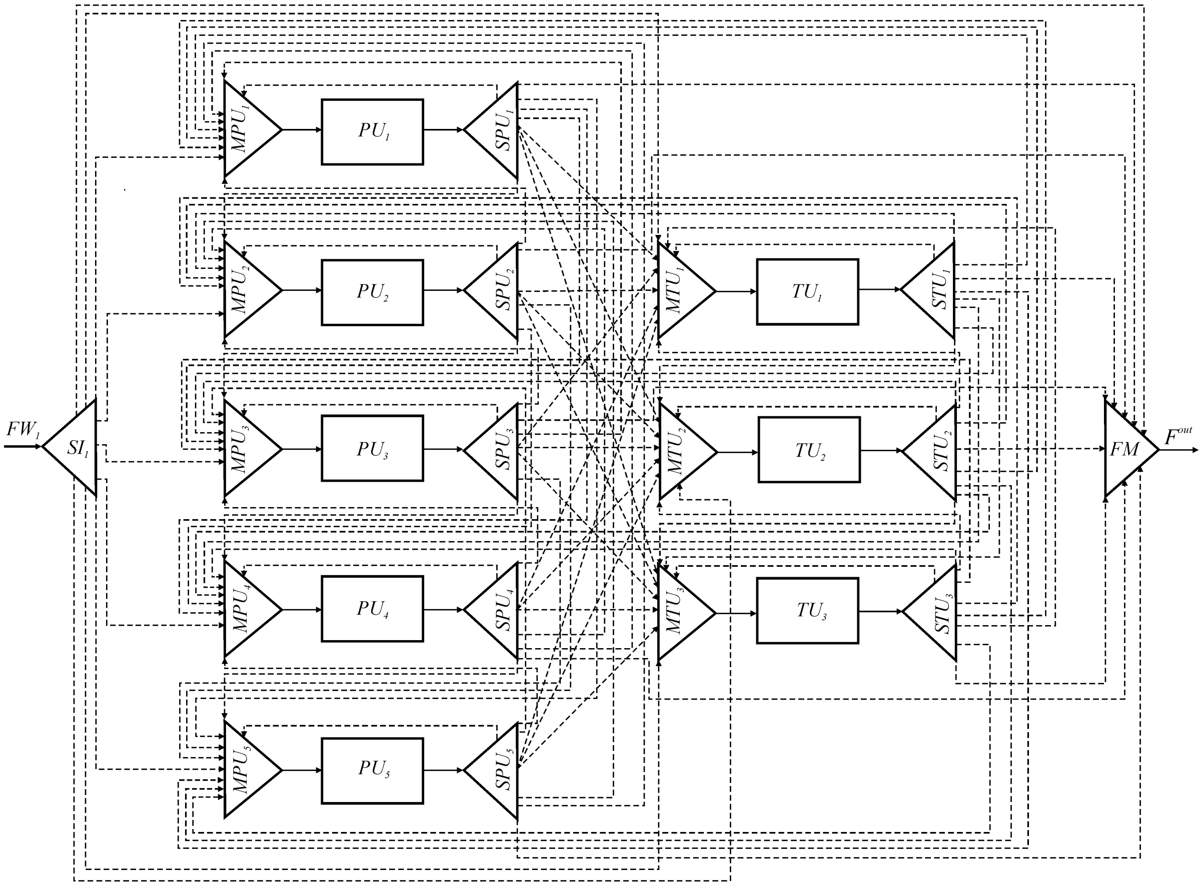

Figure 4. Water network superstucture for the example problem 2.

The input data of this example is given in Tables 4 and 5. Each treatment unit can remove only one contaminant. All other data for this example problem are the same as data given for the example problem 1. The tolerance selected for the optimization with BARON and LindoGlobal was 0.01.

Table 4. Data for process units for the example problem 2.

Process unit	Flowrate (t/h)	Discharge load (kg/h)			Maximum inlet concentration (ppm)		
		A	B	C	A	B	C
PU_1	40	1	1.5	1	0	0	0
PU_2	50	1	1	1	50	50	50
PU_3	60	1	1	1	50	50	50
PU_4	70	2	2	2	50	50	50
PU_5	80	1	1	0	25	25	25

Table 5. Data for treatment units for the example problem 2.

Treatment unit	% removal of contaminant			IC (Investment cost coefficient)	OC (Operating cost coefficient)	α
	A	B	C			
TU_1	95	0	0	16,800	1	0.7
TU_2	0	0	95	9,500	0.04	0.7
TU_3	0	95	0	12,600	0.0067	0.7

The non-convex MINLP formulation for this example contains 250 constraints, 72 binary variables, and 233 continuous variables. We similarly solved this problem with the non-convex MINLP model directly by previously mentioned solvers in GAMS on the same computer. The optimal objective function values and the computational times are given in Table 6.

Table 6. Computational performance of the Non-convex MINLP model for the example problem 2.

Solver	without ε		with $\varepsilon = 0.001$		Error (%)
	Objective function	CPU (s)	Objective function	CPU (s)	
BARON	LB: 724913 UB: 5204630	> 1000	1062733.395	95.30	$8.34 \cdot 10^{-4}$
LindoGlobal	LB: 1043010 UB: 1048254	> 1000	LB: 1043869 UB: 1552540	> 1000	-

As can be seen from the results in Table 6, this example problem cannot be solved directly by BARON and LindoGlobal solvers in reasonable computational time. However, with added tolerance $\varepsilon = 0.001$ in the concave cost terms of the objective function the derivatives become bounded, and it helps BARON to solve this example in reasonable computational time (95.30CPUs). The optimal water network design for this example in given in Figure 5.

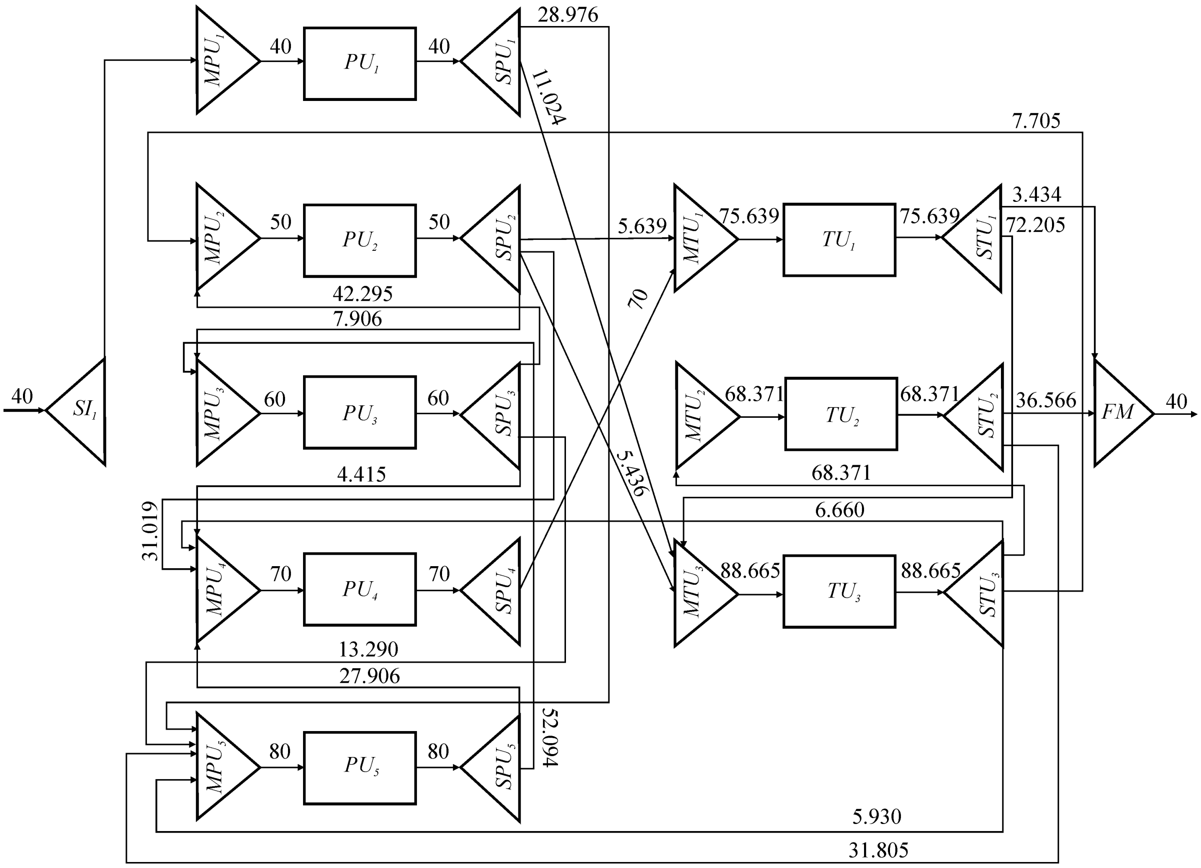

Figure 5. Optimal design of the integrated water network for the example problem 2.