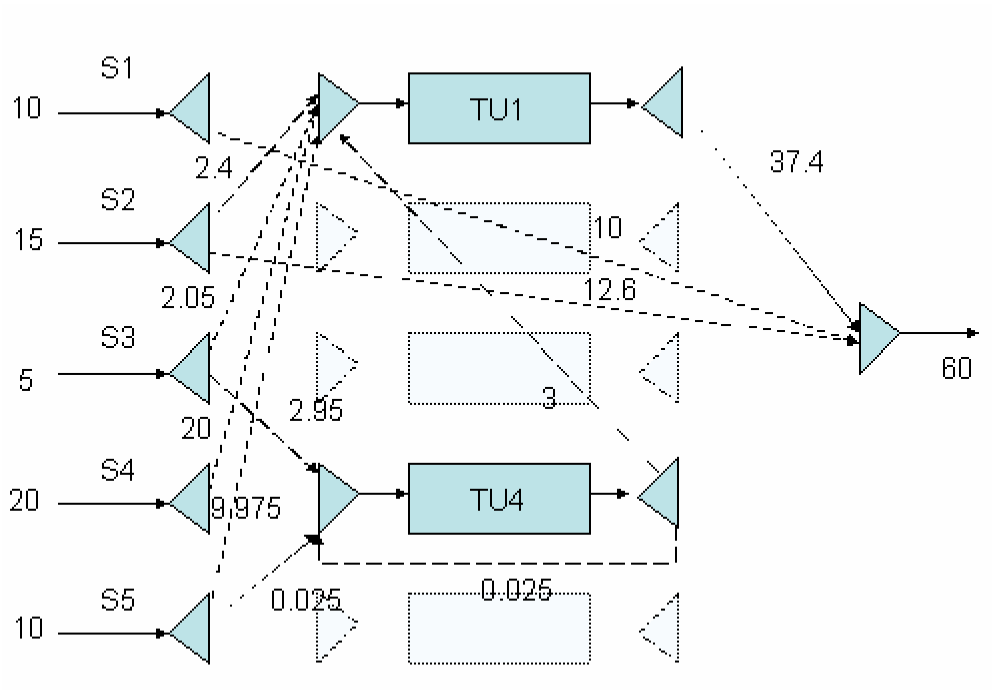
Results and Discussion

The solution of the problem is given below in which the flows of the streams in (kg/hr) are shown. Notice that only two treatment units were selected. The optimal cost of the network is $348,340



Computational Performance

To solve the particular instance for both models we used BARON (version 8.1.4) with the default options in a Pentium (R), 3.40 GHz. The size of the models is shown below:

	No 0-1 variables	No Variables	No Constraints	Non-Convex terms
Model I	5	421	380	130
Model II	5	361	320	155

We first tried to solve the problem without providing explicit bounds on the variables. In the following table the number of nodes, the lower bound at the root node and the time the solver requires is reported.

	LB	OBJ	CPU (sec)	Nodes
Conc Based F.	288791	348340	>1000	-
Flow Based F.	291157	348340	>1000	-

In order to improve the efficiency we provided bounds for the flows and concentrations by doing a physical analysis (see GAMS files). This led to the following results.

	LB	OBJ	CPU (sec)	Nodes
Conc Based F.	288791	348340	132	726
Flow Based F.	291157	348340	126	1517

Conclusion and comments:

For this problem, when suitable bounds are provided for the variables, the flow based formulation and concentration based formulation leads to similar performances, however, as it is commented below different approaches will have a great impact under slightly different conditions. In summary, it is important to consider the following:

1- When supplying bounds for the variables, these can be estimated more accurately in the case of the concentration based formulation considering the physical properties of the system. For example, a bound on concentrations can be established as the maximum concentration in the inlet streams. This ultimately will lead to tighter relaxations and hence improvements in the performance when it is solved.

2- The structure of this problem leads to two formulations with similar number of discrete and continuous variables, constraints and number and type of non-convex terms. It is clear that the non-convexities (i.e. bilinear terms) in the concentration based formulation arise in the mixer model and the ones in the flow based formulations in the splitter models. However, since the number of splitters and mixers and the number of streams going in and out are the same the number of non-convex terms are the same. A slight change in the model, for example, by considering the inlet flows of contaminated streams as variables, leads to a significant increase in the number of non-convex terms in the flow based formulation.

Data

The problem considered consists of 5 inlet streams with 4 contaminants and 4 treatment units. The data used for the test problems in both alternative formulations are given below.

Component flow j of raw contaminated streams entering splitters i

i \ j	**1**	**2**	**3**	**4**	**water**
1	1	1	1	1	10
2	2	0	0	1	15
3	1	0	1	2	5
4	3	1	1	0	20
5	2	2	0	0	10

Performance coefficient for each component j and unit k

k \ j	**1**	**2**	**3**	**4**
1	0.95	0.95	0.60	0.00
2	0.80	0.80	0.60	0.00
3	0.00	0.60	0.95	0.95
4	0.00	0.60	0.80	0.85
5	0.00	0.60	0.85	0.80

Minimum flow limit L_k for equipments k

k	*L*
1	5
2	3
3	4
4	3
5	1

Maximum contaminant flow limit in the outlet

j	*T*
1	3
2	3
3	3
4	3
5	**3**

Cost parameters for unit k

k	θ	β	γ
1	1500	8000	0
2	1000	8000	0
3	4000	8000	0
4	3000	8000	0
5	3000	8000	0