Results and Discussion

Fixed Element Models

The discretized MPCC was solved using the NLP penalty reformulation with CONOPT using $\rho =$ 1000). The results are shown in Table 1. It can be seen that the number of iterations increases almost linearly with the number of finite elements and this behavior is typical for active set strategies. In addition, the computational time per iteration increases linearly with the number of finite elements, as the majority of CPU time is expected to be spent in the sparse linear solver. In this case, sparse linear solvers increase approximately linearly in computation time with problem size. Consequently, the total computational time for CONOPT grows approximately quadratically with problem size. In addition, SBB, a simple branch and bound solver in GAMS, was applied to the above MIQP formulation of this problem. From the results in Table 1, we can observe the NP-hard aspect of solving the MIQP with this approach. Solving the problem with 1000 finite elements requires over 400 CPUs and the problem with 2000 finite elements cannot be solved within 6000 CPUs.

Variable Element Models

The discretized formulations were solved with results shown in Table 2. Again, it can be seen that the number of iterations increases almost linearly with the number of finite elements and this behavior is typical for active set strategies. In addition, the computational time per iteration increases nearly linearly with the number of finite elements, and the total computational time for CONOPT grows approximately quadratically with problem size. To solve the MINLP formulation both SBB and DICOPT, an outer approximation method that uses CONOPT and CPLEX for the NLP and MILP subproblems, respectively, were applied. From the results in Table 2 we can observe the NP-hard aspect of solving the MINLP with this approach. Solving the problem with 1000 finite elements requires well over a CPU hour and the problem with 2000 finite elements cannot be solved within 10000 CPUs.

Interpretation of Solution

The analytic solution for the hybrid dynamic system is plotted in Figure 1. Starting from $x(0) = -2$,

	Objective	SBB	CONOPT	
N	Function	CPU s.	CPU s.	Iterations
10	1.4738	0.203	0.047	14
100	1.7536	12.453	0.156	76
1000	1.7864	> 10000	7.188	675
2000	1.7886	—	25.531	1331
3000	1.7894	—	79.719	2008
4000	1.7892	—	158.797	2665
5000	1.7895	—	252.203	3325
6000	1.7898	—	361.672	3984
7000	1.7896	—	493.563	4637
8000	1.7898	—	658.156	5295

Table 1: Solution times (Pentium 4, 1.8 GHz, 992 MB RAM) comparing MIQP and MPCC formulations for fixed element models

	MINLP Formulation			MPCC Formulation		
N	Objective	SBB (CPU s.)	DICOPT (CPU s.)	Objective	Iters.	CPU s.
10	1.5364	0.188	0.905	1.5364	25	0.063
100	1.7581	58.469	> 10000	1.7766	97	0.766
1000	—	> 10000	—	1.7889	698	23.266
2000	—	—	—	1.7895	1345	77.188
3000	—	—	—	1.7897	2009	166.781
4000	—	—	—	1.7898	2705	343.016

Table 2: Solution times (Pentium 4, 1.8 GHz, 992 MB RAM) comparing MINLP and MPCC formulations for variable element models

$x(t)$ is piecewise linear and the influence of $sgn(x)$ can be seen clearly from the plot of dx/dt. Moreover, from the analytic solution, it can be shown that $x(t)$ and the objective function, ϕ are differentiable in $x(0)$. On the other hand, a discretization with fixed finite elements does not locate the transition point accurately, leading to inaccurate profiles for $x(t)$ and $\nu(t)$. In addition, this also leads to a nonsmooth dependence on $x(0)$, as seen in Figure 2. This plot with $N = 100$ shows a sawtooth behavior of ϕ vs. $x(0)$ when the elements remain fixed. In contrast with variable finite elements the objective function varies smoothly with $x(0)$. This occurs because the NLP solver now has the possibility to locate the switching points accurately, and the complementarity formulation requires the differential state $x(t)$ to remain smooth within an element. Moreover, as the Euler discretization has local errors that are $O(h_i^2)$, the piecewise linear $x(t)$ profile is captured exactly by the variable element formulation, and thus varies smoothly with $x(0)$. Nevertheless, Figure 2 shows that the plot of the *analytically determined* objective function wrt $x(0)$ still differs from the Euler discretization, despite the accurate determination of $x(t)$. This interesting difference results from $O(h^2)$ errors in the Euler approximation to the nonlinear integral in the objective function. (These errors can be eliminated by using a higher order discretization.) Note, however, that smoothness is still maintained because the integral is evaluated over the piecewise smooth portions, and not across the discontinuity.

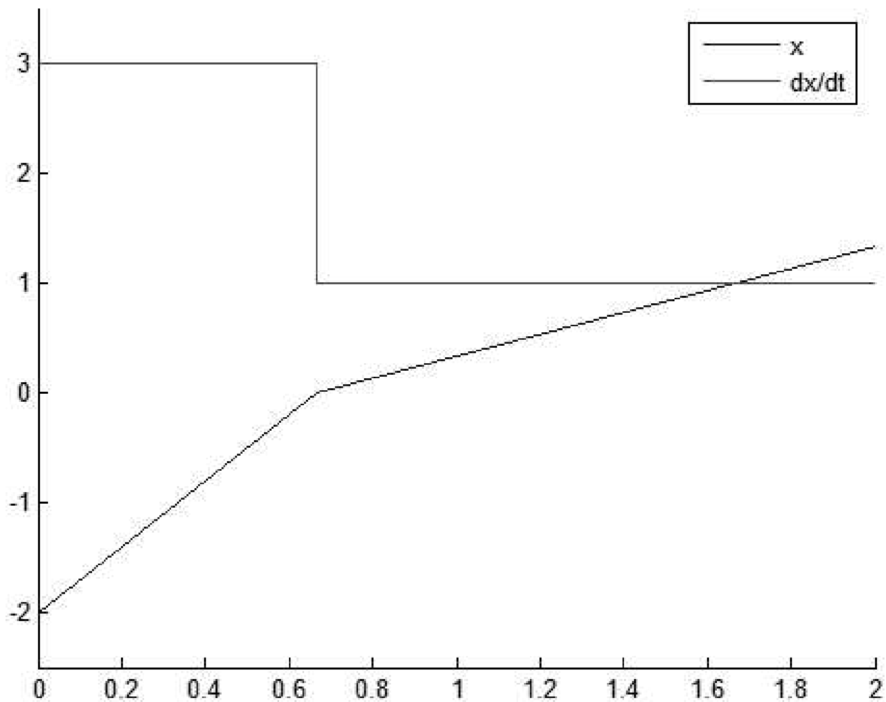

Figure 1: Solution Profiles of Hybrid Dynamic System

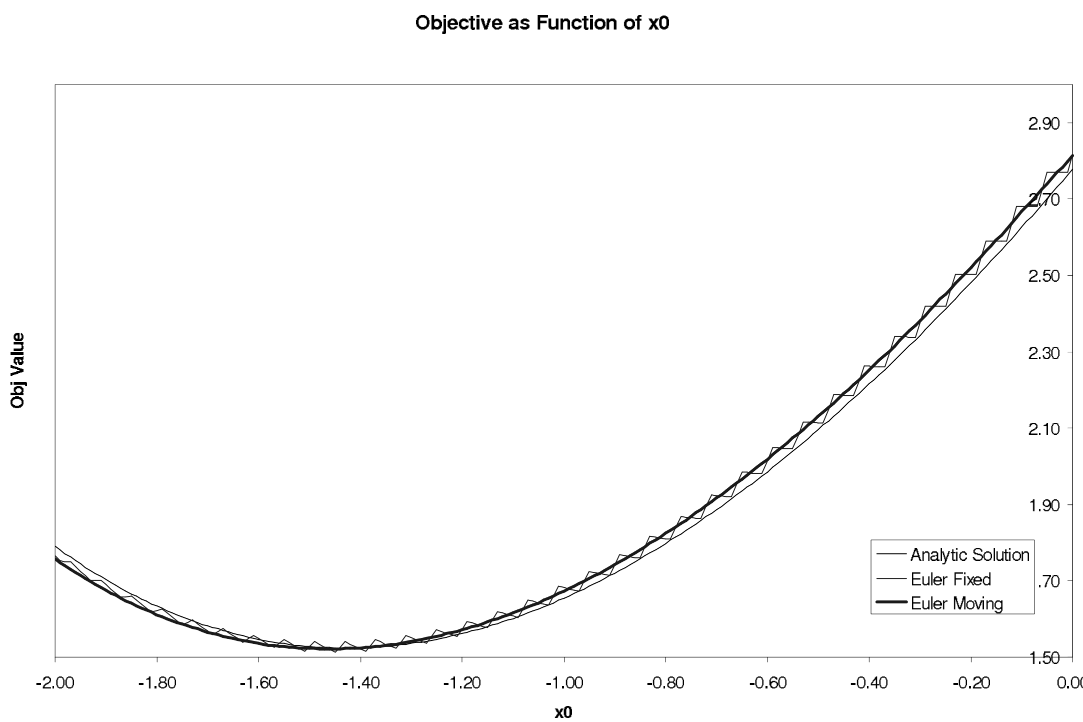

Figure 2: Sensitivity of Objective Function with Respect to x_0.