Results and Discussions

The application of this model is illustrated with the data file for a small example with one supplier, three potential DCs and six retailers as given in Figure 1.

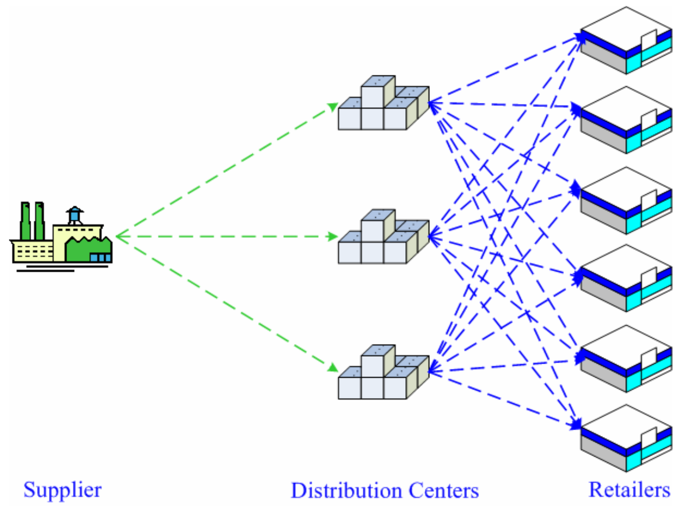


Figure 1 Supply chain network superstructure for the illustrative example

Computational Performance

We solve the problem with model (**P1**) and (**P2**) directly to obtain the global optimum (0% optimality tolerance) by using the BARON solver with GAMS on an Intel 3.2 GHz machine with 512 MB RAM. We first consider a base case with the transportation cost and inventory cost weighted factor as $\beta = 0.01$, $\theta = 0.01$, and then consider different values of weighted factors. Each instance of model (**P1**) includes 21 binary variables, 6 continuous variables, 27 constraints and 27 nonlinear nonzeros. Each instance of model (**P2**) includes 3 binary variables, 24 continuous variables, 30 constraints and 6 nonlinear nonzeros. For all the instances, global optimal solutions are obtained in less than one minute. The detailed optimal objective function value and the computational time of each instance are given in Table 1. The number of nodes required to solve (**P1**) and (**P2**), together with the objective function values for the NLP relaxations are given in Table 2. From the comparison, we can see that models (**P1**) and (**P2**) have the same global optimal solution and objective function values, but model formulation (**P2**) is much more computationally efficient than (**P1**). As seen in Table 1 model (P2) requires substantially less time. This is explained by the results in Table 2 that show that the NLP relaxation of model (P2) has zero gap, and in fact is solved at the root node. In contrast, model (P1) has a weaker relaxation and requires a substantial number of nodes.

Table 1 Computational performance of model formulation (P1) and (P2)

Transportation cost weigh factor (β)	Inventory cost weigh factor (θ)	Model (**P1**) Obj. Fu	CPU (:	Model (**P2**) Obj. Fu	CPU (:
0.01	0.01	2260.2(	7.940	2260.2(	0.470
0.1	0.01	8122.9:	28.61(	8122.9:	0.120
0.001	0.01	1099.2!	2.720	1099.2!	0.200
0.01	0.1	5359.1!	9.300	5359.1!	0.440
0.01	0.001	1341.0	7.450	1341.0	0.140

Table 2 Computational performance of model formulation (P1) and (P2)

Transportation cc weight factor (β	Inventory cost weigh factor (θ)	Model (**P1**) NLP Relaxation	# of No	Model (**P2**) NLP Relaxation	# of Nodes
0.01	0.01	2195.53	713	2260.26	1
0.1	0.01	8122.93	1162	8122.93	1
0.001	0.01	879.11	269	1099.25	1
0.01	0.1	5027.08	54	5359.18	1
0.01	0.001	1338.95	61	1341.04	1

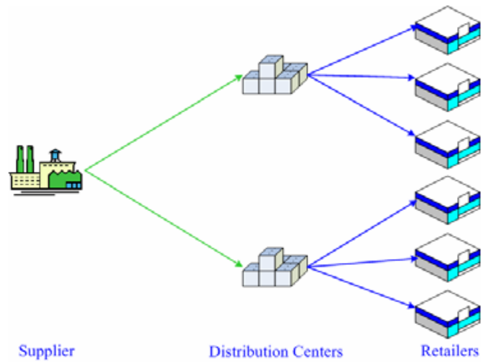
Result Discussions

The results for the example are summarized in Table 3, and their associated optimal supply chain network structures are given in Figure 2.

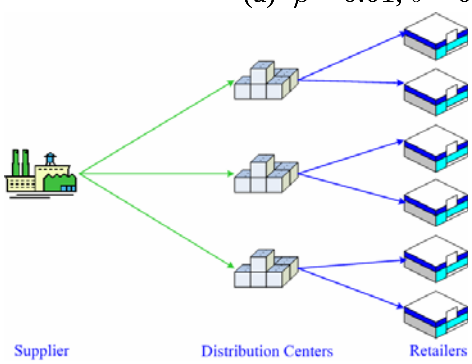
Table 3 Comparison result for the example

Transportation cost weight factor (β)	Inventory cost weigl factor (θ)	Objective Function (*Cost*)	No. DCs	Network structure
0.01	0.01	2260.26	2	Figure 6a
0.1	0.01	8122.93	3	Figure 6b
0.001	0.01	1099.25	1	Figure 6c
0.01	0.1	5359.18	1	Figure 6d
0.01	0.001	1341.04	3	Figure 6e

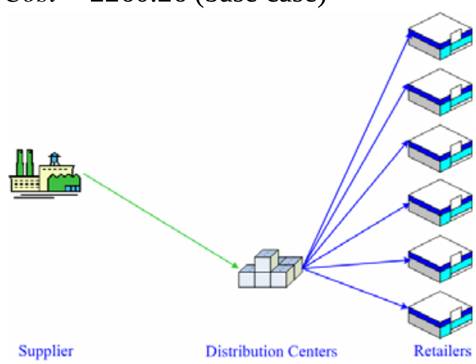
We can see that in the base case, only two DCs are selected to install and they are connected to three retailers respectively. When we increase the transportation cost factor to $\beta = 0.1$, all the three DCs are installed and each of them serves two retailers (Figure 2b). If we decrease the transportation cost factor to $\beta = 0.001$, only DC 3 is selected to install and it serves all the retailers. Thus, the larger the weighted factor for transportation costs β, the more DCs are installed. On the other hand, when we fix the transportation cost factor $\beta = 0.01$, and consider different values of the inventory cost factor θ, we can similarly find out from Figure 2d and 2e that the larger weighted factor for inventory costs, the fewer DCs are installed. It is worth mentioning that the network structures in Figure 6c and 6d both have only one distribution center installed, but the installed DC in each case is different.



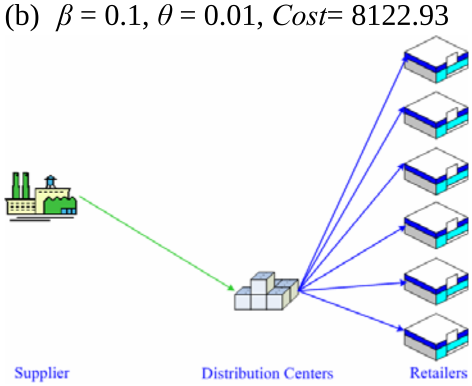
(a) $\beta = 0.01$, $\theta = 0.01$, $Cost = 2260.26$ (base case)



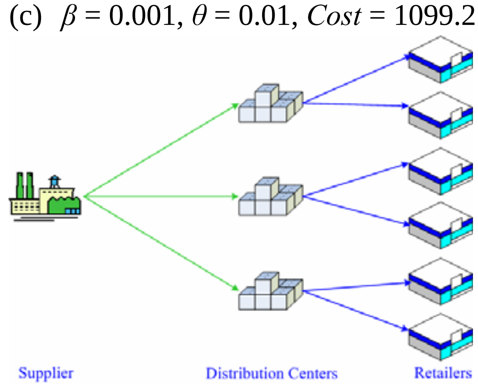
(b) $\beta = 0.1$, $\theta = 0.01$, $Cost = 8122.93$



(c) $\beta = 0.001$, $\theta = 0.01$, $Cost = 1099.25$



(d) $\beta = 0.01$, $\theta = 0.1$, $Cost = 5359.18$



(e) $\beta = 0.01$, $\theta = 0.001$, $Cost = 1341.04$

Figure 2 Optimal supply chain network structure of the illustrative example for different transportation cost and inventory cost weighted parameters

Based on this analysis, we can conclude that the more DCs are installed, the more transportation costs are potentially reduced, but less inventory cost savings are achieved. The major reason of this performance is that from an inventory cost aspect, the more retailers are pooled to a DC, the more cost can be saved, but from a transportation cost viewpoint, installing more DC to serve different retailers may reduce the total transportation costs. Thus, the trade-off between inventory and transportation costs is established and reflects on the number of DCs besides the tradeoffs for supply chain design costs and operation costs.

Appendix. Data for the Example Problem

Consider an example with one supplier, three potential distribution centers (DCs) and six retailers (i.e., i = 1,2,3,4,5,6, and j = 1,2,3). The superstructure is given in Figure A1. All the data are listed in Tables A1-A5.

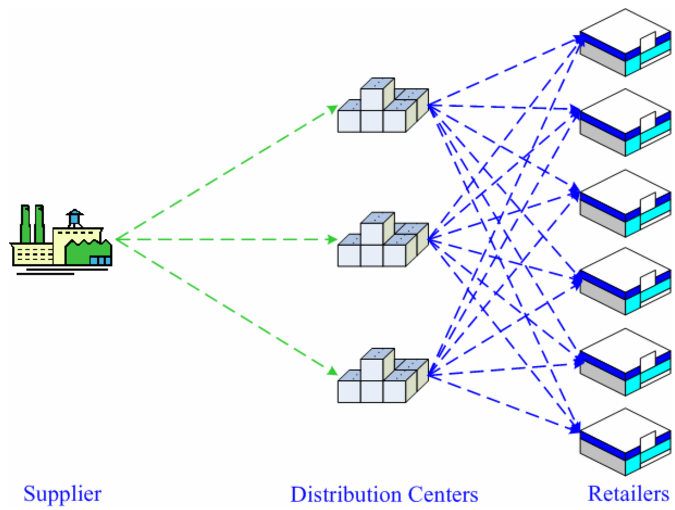


Figure A1. Supply chain network superstructure for the example

Table A1 Model coefficients for the example

F_j	Fixed order cost	10
f_j	Fixed cost to install a DC	100
z_α	Service level parameter	1.96
L	Order lead time (days)	7
h	Unit annual inventory holding cost	12
χ	Days per year	250

Table A2 Parameters for demand uncertainty

	Mean demand μ_i	Standard Deviation σ_i
Retailer 1	95	30
Retailer 2	157	50
Retailer 3	46	25
Retailer 4	234	80
Retailer 5	75	25
Retailer 6	192	80

Table A3 Parameters for unit transportation cost (d_{ij}) between retailers and DCs

	DC 1	DC 2	DC 3
Retailer 1	0.04	2.00	2.88
Retailer 2	0.08	1.36	1.32
Retailer 3	0.36	0.08	1.04
Retailer 4	0.88	0.10	0.52
Retailer 5	1.52	1.80	0.12
Retailer 6	3.36	2.28	0.08

Table A4 Parameters for fixed shipping and installation cost (f_j) of DCs

	Fixed shipping cost from supplier to DC (g_j	Unit shipping cost (a_j)
DC 1	13	0.24
DC 2	10	0.20
DC 3	14	0.28

Table A5 Weighted factors for transportation and inventory

Transportation cost weight factor (β)	Inventory cost weight factor (θ)
0.01	0.01
0.1	0.01
0.001	0.01
0.01	0.1
0.01	0.001