Optimal Periodic Scheduling of Continuous Multiproduct Plants

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Results & Discussion

Motivating Example

The motivating example (EX1) is essentially example 2 of Pinto & Grossmann (CACE 1994, 18, 797) and consists of a two stage, three products problem. The data is given in Table 1 to Table 3.

Table 1. Product Related Data for EX1

Product *i*	price, c_i (\$/kg)	minimum demand, d_i (kg/h)	final inventory cost, cf_i (\$/kg/h)
A	0.15	50	0.00406
B	0.4	100	0.00406
C	0.65	250	0.00406

Table 2. Product and Unit/Stage Related Data for EX1

unit/stage	*m*=1	*m*=2	*k*=1
product *i*	maximum processing rate, $\rho_{i,m}^{max}$ (kg/h)		intermediate inventory cost, $ci_{i,k}$ (\$/kg)
A	800	900	0.1406
B	1200	600	0.1406
C	1000	1100	0.1406

Table 3. Sequence Dependent Changeover Times, $cl_{i,i',m}$ (h) and Costs $ct_{k,i,i'}$ (\$/h) for EX1

unit/stage	*m*=1			*m*=2			*k*=1			*k*=2		
product *i/i'*	A	B	C	A	B	C	A	B	C	A	B	C
A	-	5	8	-	11	1	-	760	760	-	760	760
B	10	-	3	2	-	5	750	-	750	750	-	750
C	4	7	-	6	1	-	770	770	-	770	770	-

The best solution was found by STG for 11 event points. It features an optimal profit of 355.09 \$/h for a cycle time of 184.646 h, which is better than 352.93 \$/h. The one found by MTG, |T|=3, has a slightly lower profit 352.45 \$/h (0.7% less), and a lower cycle time, 173.519 h. The schedules and storage profiles are given in Figure 1. The maximum inventory points are slightly lower for MTG due to the lower cycle time, translating into lower intermediate and final inventory costs. The trade-off, is that a lower cycle time also means higher changeover costs since more cycles and changeovers are required on a yearly basis. STG has better product revenue due to a higher delivery rate of C, 789.18 vs. 785.01 kg/h

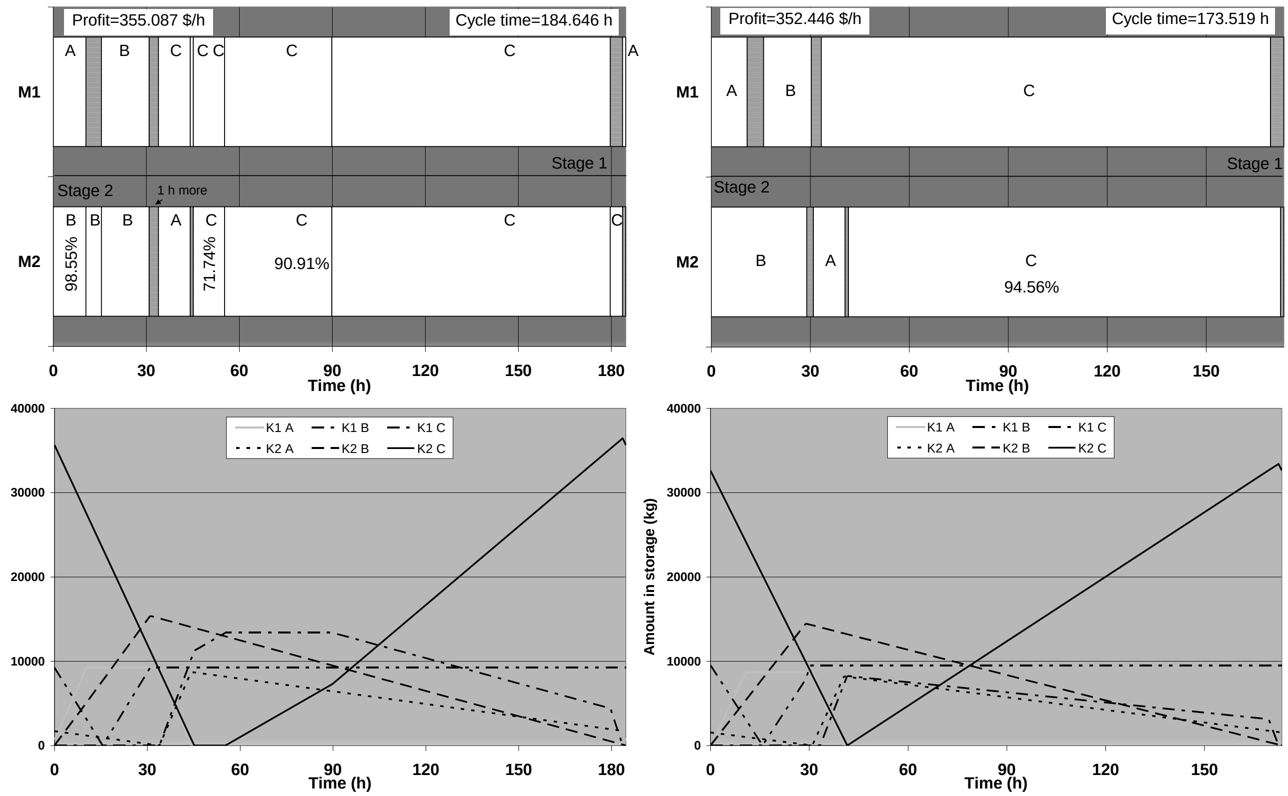

Figure 1. Best solution from models STG (left) and MTG (right) for EX1

Variants of motivating Example

EX1a,b,c allocate an extra equipment unit to stage 1, 2 and to both stages, respectively (see data in Table 4). In Figure 1 unit M1 is at its maximum processing rate, suggesting that stage 1 is the limiting one. For Ex1a the best solution is obtained by solving MTG for a profit of 391.61 $/h (10.3% increase) and a cycle time of 146.18 h. However, if the extra unit is allocated to stage 2 (EX1b) the profit raises to 464.304 $/h, found by STG. The higher profit is essentially because a reduction in final inventory cost is more important than in intermediate inventory cost. The extra unit makes it possible to use a processing rate for C equal to its delivery/demand rate. Finally, in EX1c the profit is almost doubled (687.017 $/h) when compared to the base case. This value was found by STG.

Table 4. Problem data for additional unit(s), examples EX1a,b,c

Parameter	$\rho_{i,m}^{\max}$ (kg/h)	$cl_{i,i',m}$ (h)		
product *i*/*i*'		A	B	C
A	400	-	9	11
B	500	5	-	7
C	450	6	4	-

Computational Results

The performance of models STG and MTG is illustrated through the solution of a few example problems, where most of the data was taken from Pinto & Grossmann (CACE 1994, 18, 797). They were implemented and solved in GAMS build 22.8. DICOPT was used as the MINLP solver, which in turn relied on CONOPT as the NLP solver and CPLEX as the MILP solver. The computer consisted on a Intel Core2 Duo T9300 processor (2.5 GHz) with 4 GB of RAM, running Windows Vista Enterprise. Note that in Castro and Novais (2007), slightly different solutions (some better others worse) were found for GAMS 22.2.

It is important to note that both formulations rely on a single starting point for the solution of the MINLP. For STG, all variables are at zero except the cycle time H, which is at its lower bound. For MTG, the initialization consists on having the delivery rate D_i, the processing times $\Delta T_{i,k}$ and H at their lower bounds and the processing rates $\rho_{i,k}$ at their upper bounds, and all other variables at zero. From these points, DICOPT could always find very good solutions to the problems so the whole solution process is robust. Another relevant issue is that STG is a more general formulation than MTG and so can theoretically find better solutions to the problem. In other words, global optimal solutions from MTG can be worse than local optimal solutions from STG.

Table 5. Results Overview (DICOPT solver)

						STG			MTG		
problem	products	stages	units	H^{min} (h)	H^{max} (h)	|T|	profit ($/h)	H (h)	|T|	profit ($/h)	H (h)
EX1	3	2	2	100	250	11	**355.087**	184.646	3	352.446	173.519
EX1a	3	2	3	100	250	8	390.966	148.117	3	**391.613**	146.180
EX1b	3	2	3	200	600	9	**464.304**	582.782	3	450.548	501.315
EX1c	3	2	4	200	600	7	**687.017**	396.040	3	683.971	344.975
EX2	4	2	2	400	800	9	7074.61	682.283	4	**7099.17**	674.335
EX5	3	3	3	200	600	8	5789.04	407.15	3	**5924.20**	418.800
EX6	4	3	3	200	600	8	5165.46	503.684	4	**5312.52**	478.093

The examples gradually increase in size, either due to an increase in the number of products or in the number of stages, with examples EX2, 5-6 featuring a single unit per stage. The most important result in Table 5 is that the single time grid formulation leads to the best solutions (in bold) only for EX1, EX1b and EX1c. This behavior is entirely due to two performance factors since in the limit of an infinite number of event points, the feasible region from STG includes that of MTG. The first performance factor, is that the STG generates harder MINLPs (see Table 6), due to the use of more event points. Finding the optimal solution involves iterating on |T| and since a single increment may involve an increase in computational effort up to one order of magnitude, the mathematical problems rapidly become intractable, probably before reaching the value of |T| that would allow generating the same solution as that from the MTG. The second, is that the problem is non-convex and local optimization solvers may fail to find the global optimal solution for a given |T| value. This is particular important since it also affects the search procedure over |T|. More specifically, we have found that a single increase in |T| sometimes led to a worse solution despite the fact that the feasible region had increased.

Table 6. Overview of Computational Statistics (DICOPT solver)

	STG				MTG			
problem	binary variables	continuous variables	constraints	CPU time (s)	CPU time (s)	binary variables	continuous variables	constraints
EX1	198	415	262	21.8	1.68	48	125	196
EX1a	216	424	250	34.1	1.68	93	194	257
EX1b	243	476	280	37.4	1.53	93	194	257
EX1c	252	477	270	33.5	2.23	120	245	319
EX2	288	523	280	70.5	1.19	112	230	307
EX5	216	451	299	25.3	1.92	78	191	309
EX6	384	694	388	95.3	62.9	176	350	481