A Deterministic
Security Constrained Unit Commitment Model

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Modeling data

For simulation purposes data was taken from a security constrained unit commitment problem (SCUC) from (Niknam, Khodaei et al. 2009). The basic problem consists of 10 units with a scheduling time horizon of 24 h. The spinning reserve is fixed at 10% of the scaled demand load. Table 1 and Table 2 give the demand and the data of the generator units, respectively.

Table 1: Load demand

Hour	D_t (MW)
1	700
2	750
3	850
4	950
5	1000
6	1100
7	1150
8	1200
9	1300
10	1400
11	1450
12	1500
13	1400
14	1300
15	1200
16	1050
17	1000
18	1100
19	1200
20	1400
21	1300
22	1100
23	900
24	800

Table 2: System data

Unit	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW^{2}h)	EC_i (g/kWh)	p_i^U (MW)	p_i^L (MW)
1	1000	16.19	0.00048	500	455	150
2	970	17.26	0.00031	500	455	150
3	700	16.60	0.00200	500	130	20
4	680	16.50	0.00211	500	130	20
5	450	19.70	000398	500	162	25
6	370	22.26	0.00712	500	80	20
7	480	27.74	0.00079	780	85	25
8	660	25.92	0.00413	780	55	10
9	665	27.27	0.00222	780	55	10
10	670	27.79	0.00173	780	55	10

Unit	RU_i (MW/h)	RD_i (MW/h)	SU_i (\$/h)	SD_i (\$/h)	TU_i (h)	TD_i (h)
1	200	200	10	10	8	8
2	200	200	10	10	8	8
3	100	100	8	8	5	5
4	100	100	8	8	5	5
5	100	100	8	8	6	6
6	50	50	10	10	3	3
7	50	50	10	10	3	3
8	50	50	8	8	1	1
9	50	50	8	8	1	1
10	50	50	8	8	1	1

Results

The network of 10 generators that should be committed over a horizon of 24 hours contains 5.330 Equations, 961 Variables (240 Nonlinear and 720 Discrete). The problem is convex. Several solvers available within GAMS were tested and the results are summarized below. Figure 1 shows the optimized power profiles for each generator.

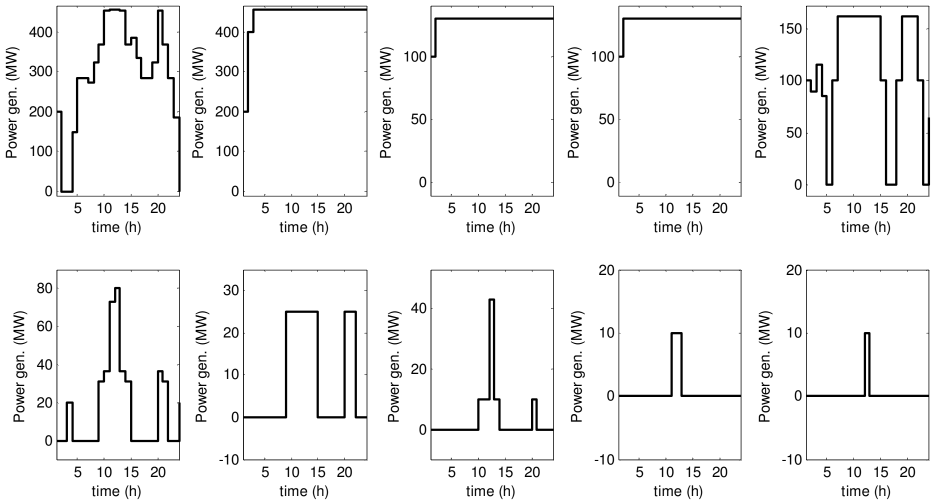

Figure 1: Optimized power production profiles for each generator

Computational Results

We have compared the numerical results for two formulations with three MINLP solvers: DICOPT, SBB and BARON. The first model corresponds to the convex MIQP presented in section 3. In the second model the objective function contains a product of a discrete and a continuous variable, resulting in the traditional non-convex problem. Table 3 shows the results for the two models.

Table 3: Comparison of convex and non-convex models with different solvers * *CPU time in seconds*

MODEL	**SBB**		**DICOPT**		**BARON**	
	*CPU time**	*Nodes*	*CPU time**	*Maj. It.*	*CPU time**	*BaR Nodes*
Convex MINLP	3600	30566	2.7	3	3600	505
	Obj. val. 586979 (found at node 28801)		578176		Obj. val. 581427	

As can be seen in Table 3, DICOPT is the fastest in solving the requiring only 2.7 sec and finding the global optimum of 578176. In contrast, SBB and BARON are unable to find the global optimum for the convex case as they reach the time limit of 3600 sec. The best possible solution found for SBB produces a higher value of 586979, which was found in node 28801 out of a total of 30566 nodes in the branch and bound search tree. BARON obtains a slightly lower value of 581427.

Instead of using general MINLP solvers, the convex MIQCP can also be solved with CPLEX. This results in a further reduction of computational time, as shown in Table 4. While DICOPT requires a CPU time of 2.8 sec, CPLEX solves the problem in 0.7 sec. This can in part be attributed to the small relaxation gap of the MIQP problem (1.6%). Compared to any of the SBB and BARON solvers, it is clear that solving the

MIQP with CPLEX can be solved orders of magnitudes faster. Compared to DICOPT the reduction is less dramatic.

Table 4: Comparison of the nonlinear and quadratic form of the unit commitment problem

DICOPT (MINLP)		**CPLEX (MIQCP)**	
CPU time*	_Iterations_	_CPU time*_	_Iterations_
2.8	1287 3 Maj. It.	0.7	1670 5 Nodes
Obj. value 578176		Obj. value 578176	
Relaxation gap 1.6%		Relaxation gap 1.6%	

References

Niknam, T., Khodaei A., Fallahi, F.. (2009). "A new decomposition approach for the thermal unit commitment problem." Applied Energy 86(9) pp.1667-1674.