Results and Discussions

The application of this model is illustrated first with a small example problem 1 for a plant with 5 products, 6 processing stages and a maximum of 4 processing units per stage. The input data of this example is given in Table 1.

We solved the problem with the non-convex and convex MINLP models directly by using the Alpha-ECP 1.75.03, BARON 8.1.5, Bonmin 1.0, DICOPT 23.2.1 and SBB solvers with GAMS 23.2.1 on an Intel 3.2 GHz machine with 2 GB RAM. In the non-convex only two 0-1 variables are needed per stage due to the use of binary expansions for the number of processing units in parallel. Hence, the non-convex MINLP formulation contains 12 binary variables and 22 continuous variables. The objective function and 31 of the 67 constraints in the model are nonlinear. With the convex MINLP formulation, there are 24 binary variables and 22 continuous variables. Only the objective function and 1 of the 73 constraints are nonlinear. The initial values of all the variables are set to be on their lower bound, although other values can be specified. It is interesting to note that global optimal solutions are obtained in less than one minute for all the instances. The detailed optimal objective function value and the computational time of each instance are given in Table 2. From the comparison, we can see that both convex and nonconvex models have the same global optimal solution and objective function values, but convex model formulation is usually more computationally efficient. In addition, MINLP solver DICOPT and SBB have higher computational efficiency than other solvers for these instances. The global optimum for the two equivalent formulations has an investment cost of $285,506 and details of the optimal solution are summarized in Table 3.

Table 1. Data for the example problem 1.

	stages, j = 1, 6	products, i = 1, 5	
cost coefficient	α_j = $250	β_j = 0.6	j = 1, 6
bounds on volumes	V_j^L = 300 L	V_j^U = 3000 L	j = 1, 6
maximum number of units		N_j^U = 4	j = 1, 6
production horizon time	H = 6000 hr		

	Q_i : Fixed production target of product i (kg)			
A	B	C	D	E

250,000	150,000	180,000	160,000	120,000

	S_{ij} : size factor for product i in processing stage j (L/kg)					
product	1	2	3	4	5	6
A	7.9	2.0	5.2	4.9	6.1	4.2
B	0.7	0.8	0.9	3.4	2.1	2.5
C	0.7	2.6	1.6	3.6	3.2	2.9
D	4.7	2.3	1.6	2.7	1.2	2.5
E	1.2	3.6	2.4	4.5	1.6	2.1

	t_{ij} : processing time for product i in processing stage j (hrs)					
product	1	2	3	4	5	6
A	6.4	4.7	8.3	3.9	2.1	1.2
B	6.8	6.4	6.5	4.4	2.3	3.2
C	1.0	6.3	5.4	11.9	5.7	6.2
D	3.2	3.0	3.5	3.3	2.8	3.4
E	2.1	2.5	4.2	3.6	3.7	2.2

Table 2. Computational performance of the Non-convex and Convex MINLP model formulations for the example problem 1

Solvers	Non-convex MINLP Model		Convex MINLP Model	
	Obj. Fun.	CPU (s)	Obj. Fun.	CPU (s)
Alpha-ECP	285506.5082	4.6	285506.5082	9.1
BARON	285506.5082	23.9	285506.5082	1.3
Bonmin	285506.5082	3.8	285506.5082	4.7
DICOPT	285506.5082	0.7	285506.5082	0.2
SBB	285506.5082	0.4	285506.5082	0.6

Table 3. Optimal solution for the example problem 1.

	Product				
	A	B	C	D	E
TL_i, hr	3.2	3.4	6.2	3.4	3.7
B_i, kg	380	770	730	638	525

	Stage					
	1	2	3	4	5	6
N_j	2	2	2	2	1	1
V_j, L	3000	1892	1975	2619	2328	2110

	Y_{kj} in the non-convex MINLP formulation					
k \ stage	1	2	3	4	5	6
1	1	1	0	1	0	0
2	0	0	1	0	0	0

	Y_{kj} in the convex MINLP formulation					
k \ stage	1	2	3	4	5	6
1	0	0	0	0	1	1
2	1	1	0	1	0	0
3	0	0	1	0	0	0

4	0	0	0	0	0	0

The second example problem is for a plant with 8 products, 12 processing stages and a maximum of 5 processing units per stage. The input data of this example is given in Table 4. We similarly solved the problem with the non-convex and convex MINLP models directly with the aforementioned MINLP solvers in GAMS on the same machine. The non-convex MINLP formulation contains 36 binary variables and 40 continuous variables. The objective function and 97 of the 205 constraints in the model are nonlinear. With the convex MINLP formulation, there are 60 binary variables and 40 continuous variables. Only the objective function and 1 of the 217 constraints are nonlinear. The initial values of all the variables are also set to be on their lower bound. The computational results with detailed optimal objective function value and the CPU time of each instance are given in Table 5. From the comparison, we can see that both convex and nonconvex models have the same global optimal solution and objective function values, but convex model formulation usually requires less CPU times. In addition, MINLP solver DICOPT and SBB have higher computational efficiency than other solvers for these instances. The global optimum for the two equivalent formulations has an investment cost of $2,687,027 and details of the optimal solution are summarized in Table 6.

Table 4. Data for the example problem 2.

	stages, j = 1, 12	products, i = 1, 12	
cost coefficient	α_j = $250	β_j = 0.6	j = 1, 6
bounds on volumes	V_j^L = 300 L	V_j^U = 3000 L	j = 1, 6
maximum number of units		N_j^U = 5	j = 1, 6
production horizon time	H = 6000 hr		

α_j : cost coefficient for stage j ($)					
1	2	3	4	5	6
250	550	250	1000	300	800
7	8	9	10	11	12
200	1200	250	250	450	700

Q_i : Fixed production target of product i (kg)							
A	B	C	D	E	F	G	H
485,000	297,000	320,000	283,000	363,000	265,000	288,000	145,000

S_{ij} : size factor for product i in processing stage j (L/kg)						
product	1	2	3	4	5	6
A	7.9	2.0	5.2	4.9	6.1	4.2

B	0.7	0.8	0.9	3.4	2.1	2.5
C	0.7	2.6	1.6	3.6	3.2	2.9
D	4.7	2.3	1.6	2.7	1.2	2.5
E	1.2	3.6	2.4	4.5	1.6	2.1
F	0.7	2.4	3.1	2.2	3.7	4.8
G	2.3	4.7	5.2	3.5	2.9	3.6
H	0.4	0.9	1.1	1.4	1.6	2.2
	7	8	9	10	11	12
A	2.8	3.3	4.1	3.8	2.8	3.9
B	3.3	3.0	2.7	2.4	2.2	3.1
C	2.6	2.2	4.6	4.3	4.2	4.6
D	1.5	1.5	1.3	1.7	1.5	1.8
E	2.4	2.7	2.8	3.5	3.5	4.3
F	4.5	5.2	6.4	5.7	6.4	6.8
G	3.3	3.2	4.1	3.7	3.4	3.7
H	2.0	1.8	1.8	1.6	1.8	2.0

	t_{ij} : processing time for product i in processing stage j (hrs)					
product	1	2	3	4	5	6
A	6.4	4.7	8.3	3.9	2.1	1.2
B	6.8	6.4	6.5	4.4	2.3	3.2
C	1.0	6.3	5.4	11.9	5.7	6.2
D	3.2	3.0	3.5	3.3	2.8	3.4
E	2.1	2.5	4.2	3.6	5.7	2.2
F	1.1	0.8	0.4	1.1	1.8	2.5
G	4.2	4.0	2.2	0.5	3.4	2.2
H	2.7	4.3	1.9	2.0	1.7	0.7
	7	8	9	10	11	12
A	0.8	2.2	1.2	2.5	3.4	3.8
B	0.4	0.2	0.5	3.3	0/6	1.2
C	1.1	0.6	1.2	4.3	2.8	5.2
D	1.7	0.9	2.2	2.15	1.8	2.5
E	1.2	0.6	1.15	3.1	4.2	1.6
F	0.5	1.3	1.4	4.25	2.7	0.9
G	1.4	0.9	2.1	4.4	2.2	3.2
H	0.3	0.2	1.6	3.5	3.4	2.1

Table 5. Computational performance of the Non-convex and Convex MINLP model formulations for the example problem 2

Solvers	Non-convex MINLP Model		Convex MINLP Model	
	Obj. Fun.	CPU (s)	Obj. Fun.	CPU (s)
Alpha-ECP	2687026.78	6.8	2687026.78	5.3
BARON	2687026.78	52.97	2687026.78	4.11
Bonmin	2687026.78	27.25	2687026.78	26.515
DICOPT	2687026.78	0.5	2687026.78	1.5
SBB	2687026.78	1.2	2687026.78	12.6

Table 6. Optimal solution for the example problem 2.

	Product				
	A	B	C	D	E
TL_i, hr	1.66	1.36	2.38	0.85	1.14
B_i, kg	380	842	652	638	636
	F	G	H		
TL_i, hr	0.85	1.07	0.86		
B_i, kg	439	547	957		

	Stage					
	1	2	3	4	5	6
N_j	5	5	5	5	5	4
V_j, L	3000	2570	2843	2864	2316	2106
	7	8	9	10	11	12
N_j	2	2	3	5	4	3
V_j, L	2780	2527	3000	2804	2808	3000

	Y_{kj} in the non-convex MINLP formulation					
k \ stage	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	1
3	1	1	1	1	1	0
k \ stage	7	8	9	10	11	12
1	1	1	0	0	1	0
2	0	0	1	0	1	1
3	0	0	0	1	0	0

	Y_{kj} in the convex MINLP formulation					
k \ stage	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	1	1	1	1	1	0
k \ stage	7	8	9	10	11	12
1	0	0	0	0	0	0
2	1	1	0	0	0	0
3	0	0	1	0	0	1
4	0	0	0	0	1	0
5	0	0	0	1	0	0

Reference

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