Stabilizing controller design and the Belgian chocolate problem

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Overview

In the chemical process industries, a set of controllers are strategically placed so as to enforce certain control objectives on the underlying process. Because of its close relation to safety, stability is one of the objectives often considered. Therefore, stabilization, *i.e.* designing a controller that stabilizes an unstable process, is a central aspect in controller system design. If the control law to be applied to the system is determined beforehand, two types of stabilization problems can arise. The first one is the *stabilizability problem*, in which one must determine under what conditions the process is stabilizable with the control law. The second problem is the *stabilizing controller design problem*, in which one must find a controller accomplishing stabilization of the given process.

In the Belgian chocolate problem, a process with a single parameter δ is given, and we want to find the range of δ, for which the process is stabilizable with a minimum phase stable controller. It has been shown theoretically that the process is stabilizable if and only if $\delta < \bar{\delta}$, and $\bar{\delta}$ is slightly smaller than 1 [1]. Finding the exact value of $\bar{\delta}$ solves the stabilizability problem. This problem is currently open. However, computational approaches have reported several values of δ, for which a stabilizing controller could be found [2, 4], thus providing valid lower bounds for $\bar{\delta}$. The best such lower bound for $\bar{\delta}$ currently known is 0.974 [4].

Since the stability of a polynomial requires the maximum real part of its zeros to be negative, the stabilizing controller design problem results in a bilevel minimax optimization program. In order to transform the bilevel problem to a single-level program, the authors of [4] extended an equivalent reformulation technique that was proposed to handle dynamic stability of a matrix [3]. Here, we provide a GAMS implementation of the nonconvex MINLP for the stabilizing controller design problem for the Belgian chocolate problem using up to degree 8 controllers.

References

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