Results and discussion

The included GAMS input and output files are named `bcp#.gms` and `bcp#.lst` depending on the degree of controller running from 5 to 8. Model statistics are summarized in Table 1.

Table 1: Problem sizes

degree	equations	0-1 variables	continuous variables
5	104	6	85
6	135	7	112
7	154	8	127
8	191	9	160

The parameter values that are used in the implementation are

$$\epsilon = 0.001, \quad \tau = 10, \quad \mu = 0.00001, \quad \varepsilon = 0.00001, \quad \sigma = 10$$

for all instances. These values can be adjusted depending on the system architecture that runs the GAMS program. Users are encouraged to try different ranges of coefficients $[\epsilon, \tau]$ to see how the change affects the solution.

We used DICOPT to obtain the solutions shown in Table 2. We also attempted to solve the problems to global optimality with BARON but the problems were not solvable in 3,600 CPU seconds. The best lower bounds obtained for each n are comparable to those reported in [1] with the additional advantage of increased numerical robustness thanks to the newly introduced parameter μ.

Table 2: Best solutions obtained for problems of varying degree

degree	lower bound
5	0.9519
6	0.9609
7	0.9629
8	0.9697

For example, the degree 5 stabilizing controller corresponding to the lower bound reported in Table 2 is:

$$\begin{aligned} x(s) &= 3.539652085474288 s^5 + 6.764011230693175 s^4 \\ &\quad + 9.356032136872777 s^3 + 10 s^2 \\ &\quad + 6.182320013166775 s + 1.409062387538898 \end{aligned}$$

$$y(s) = 1.048186259969022s^2 + 3.499738483623286s + 1.409062387438883$$

Stability conditions for x and z are active for this controller as can be observed in Figure 1 that depicts the zeros of x and z on complex plane.

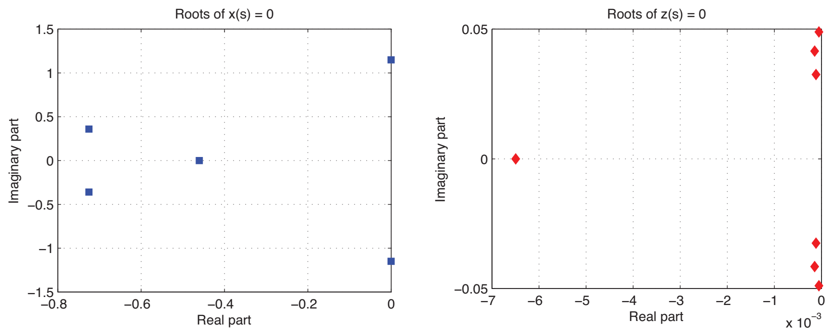

Figure 1: Roots of polynomials x and z for the degree 5 controller. Stability constraints for y are not binding, and both real roots of y are smaller than -0.4.

The provided GAMS implementations are successfully used to find lower bounds of $\bar{\delta}$ for which the process is stabilizable using controllers of different degrees. The newly introduced stability margin feature ensures that the controller found by an optimization solver indeed stabilizes its corresponding process. Notice that the included GAMS codes generate a simple Matlab file that can be used to verify if the roots of the polynomials x, y, and z are all stable. All the controllers reported in the listing files are verified to correspond to stabilizing controllers.

References

[1] Y. Chang and N. V. Sahinidis. Global optimization in stabilizing controller design. *Journal of Global Optimization.* **38**(4):509-526, 2007.