A Non-Convex Deterministic
Security Constrained Unit Commitment Model

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Modeling data

For simulation purposes data was taken from a security constrained unit commitment problem (SCUC) from (Niknam, Khodaei et al. 2009). The basic problem consists of 10 units with a scheduling time horizon of 24 h. The spinning reserve is fixed at 10% of the scaled demand load. Table and Table give the demand and the data of the generator units, respectively.

Table 1: Load demand

Hour	D_t (MW)
1	700
2	750
3	850
4	950
5	1000
6	1100
7	1150
8	1200
9	1300
10	1400
11	1450
12	1500
13	1400
14	1300
15	1200
16	1050
17	1000
18	1100
19	1200
20	1400
21	1300
22	1100
23	900
24	800

Table 2: System data

Unit	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW^{2}h)	EC_i (g/kWh)	p_i^U (MW)	p_i^L (MW)
1	1000	16.19	0.00048	500	455	150
2	970	17.26	0.00031	500	455	150
3	700	16.60	0.00200	500	130	20
4	680	16.50	0.00211	500	130	20
5	450	19.70	000398	500	162	25
6	370	22.26	0.00712	500	80	20
7	480	27.74	0.00079	780	85	25
8	660	25.92	0.00413	780	55	10
9	665	27.27	0.00222	780	55	10
10	670	27.79	0.00173	780	55	10

Unit	RU_i (MW/h)	RD_i (MW/h)	SU_i (\$/h)	SD_i (\$/h)	TU_i (h)	TD_i (h)
1	200	200	10	10	8	8
2	200	200	10	10	8	8
3	100	100	8	8	5	5
4	100	100	8	8	5	5
5	100	100	8	8	6	6
6	50	50	10	10	3	3
7	50	50	10	10	3	3
8	50	50	8	8	1	1
9	50	50	8	8	1	1
10	50	50	8	8	1	1

Results

The network of 10 generators that should be committed over a horizon of 24 hours contains 5.330 Equations, 961 Variables (240 Nonlinear and 720 Discrete).

We have compared the numerical results of two simulation sessions. In this session we have optimized the unit commitment problem while the objective function contains a product of a discrete and a continuous variable, resulting in a non-convex problem. This formulation gives complications during optimization. In the earlier session submitted model we have rewritten the problem in such way that the product of the discrete and continuous variable is no longer part of the objective function, and the overall problem becomes quadratic and convex. The table below shows the outcomes for the two sessions:

	SBB		**DICOPT**		**BARON**	
	*CPU time**	*Nodes*	*CPU time**	*Maj. It.*	*CPU time**	*BaR Nodes*
Convex	41	316	2.7	3	3600	505
	Obj. val. 591974 (found at node 83)		578176		Obj. val. 581427	
Non-convex	67	300	50.4	4	3600	3289
	Obj. val. 590521 (found at node 266)		Obj. val. 598902		Obj. val. 637832	

** CPU time is recorded in seconds*

DICOPT is the fastest in solving the convex problem, requiring only 3 major iterations and finding the global optimum of 578176. SBB is faster than BARON for the convex case, but the best possible solution found in node 83 out of a total of 266 nodes in the branch and bound search produces a slightly higher value of 591974, presumably due to some tolerance issues. BARON cannot terminate the search after 3600 sec, and obtains a value of 581427. For the non-convex formulation, none of the three solves finds the global optimum. SBB yields the best value of 590521 (interestingly better than in its convex formulation), while DICOPT yields a slightly higher value of 598902. BARON again cannot terminate the search after 3600 seconds and obtains the highest value, 637832. In terms of time, DICOPT is slightly faster than SBB (50.4 secs vs 67 secs).

From these results it is clear that the non-convex formulation is inferior to the convex MINLP, which is of course not surprising.

References

Niknam, T., A. Khodaei, et al. (2009). "A new decomposition approach for the thermal unit commitment problem." Applied Energy.